The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: MHZ5530/MPZ5230 Engineering Mathematics III**
Academic Year	: 2019/20
Date	: 9th October 2020
Time	: 0930 - 1230hrs
Duration	: **3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Nine (9)** questions in **Six (6)** pages.
3. Answer any **Six (6)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Relevant statistical tables are provided.
6. This is Closed Book Test (CBT).
7. Answers should be in clear handwriting.
8. Do not use Red colour pen.

Question 01

a) Find the value of the surface integral of the function $f(x, y) = x$, throughout the area enclosed by the curves $y = \frac{1}{2}x$ and lying between the planes $x + y = 3$ and $y = 0$.

[30 %]

b) Determine the volume of the region that lies behind the plane $x + y + z = 6$ and in front of the region in the yz plane that is bounded by $z = \frac{1}{2}\sqrt{y}$ and $z = \frac{1}{4}y$. [40 %]

c) Find the volume of the cylinder $x^2 + y^2 = 2y$, enclosed between the planes $z = 1$ for $z = 0$. [30 %]

Question 02

a) The velocity of a fluid is given by the vector field $\underline{u}(r, \theta, z) = \frac{\theta}{r}\underline{e}_r + \frac{r}{\theta}\underline{e}_\theta \quad for \ (r \neq 0)$.

Find the flux of the fluid ($\int_S \underline{u} \cdot \underline{n}\, dA$), through the curved surface S of the cylinder $r = R$ between $0 \leq z \leq 1$, where $\underline{n}$ is the unit outward normal to the surface S.

[30 %]

b) i. State the Divergence Theorem. [15 %]

ii. Verify the Divergence Theorem for the vector filed $\underline{F} = x^2\underline{i} + y^2\underline{j} + z^2\underline{k}$ on the region G given by, $0 \leq x, y, z \leq 1$. [55 %]

Question 03

a) State Stokes' Theorem. [20 %]

b) Consider the vector field $\underline{F} = (x + y)\underline{i} + (2x - z)\underline{j} + (y + z)\underline{k}$.

i. Find $curl\ \underline{F}$. [20 %]

ii. Evaluate $\oint_C \underline{F}.d\underline{r}$, for the surface ($s$) of a tetrahedron represent by the equation $x + y + z = 2$. [30 %]

iii. Verify the Stokes' Theorem by using part b) i. and part b) ii. . [30 %]

Question 04

a)

i. Define an irrotational vector field. [10 %]

ii. Write down any two properties of an irrotational vector field. [10 %]

b) Consider the vector field $\underline{F} = (y + z)\underline{i} + (x + z)\underline{j} + (x + y)\underline{k}$.

i. Show that the vector field $\underline{F}$ with domain $\mathbb{R}^3$ is an irrotational vector. [20 %]

ii. Find a scalar potential function ϕ such that $\underline{F} = grad\phi$. [30 %]

c) Evaluate, $\oint_c 2y^2dx + 3xdy$ using Green's Theorem(In Plane), where C is the boundary of the closed region bounded by $y = x$ and $x = \sqrt{y}$ in counter clockwise direction. [30 %]

Question 05

a) Find the image of the rectangular region in z plane bounded by $1 \leq x \leq 2$, and $1 \leq y \leq 2$, under the mapping $w = z^2$. [30 %]

i. Find the Machaurin series expansion of the function, $f(z) = e^z$. [30 %]

ii. Hence obtain the Machaurim series of the function $f(z) = \cosh z$. [20 %]

[*Hint:* $\cosh\theta = \frac{e^{-\theta}+e^{\theta}}{2}$]

b) Find the Laurent series around $z = 0$ for $f(z) = \frac{1}{z(z-1)}$ in each of the following region.

i. $0 < |z| < 1$ [10 %]

ii. $0 < |z| < \infty$ [10 %]

Question 06

The moment of inertia tenser is given by,

$$T_{ij} = \begin{bmatrix} a & 0 & 0 \\ 0 & 3 & -2 \\ 0 & -2 & 3 \end{bmatrix}, \text{ where } a \in \mathbb{R} \text{ and } a > 0.$$

a) Find the eigenvalues and the principle axes of the tensor in terms of a. [60 %]

b) Express the tensor with respect to its principle axes in terms of a. [20 %]

c) The transformation matrix of T is given by,

$$\bar{T} = \begin{bmatrix} 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

Express the tensor with respect to its principle axes and verify your result by using the transformation law for the second order tensor. [20 %]

Question 07

a) Apply Cosine Fourier transform to solve:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad for\ x \geq 0, t \geq 0,$$

with the initial and boundary conditions,

i. $\frac{\partial u}{\partial x}(0,t) = -2 \qquad for\ t > 0$

ii. $u(x,0) = \begin{cases} 1 & 0 < x < 1 \\ 0 & x \geq 1 \end{cases}$

$u \to 0$ uniformly in t, as $x \to \pm\infty$

$u_x \to 0$ uniformly in t, as $x \to \pm\infty$.

[50 %]

b) Solve the following initial and boundary value problem using the variable separable method.

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0, \text{ where } 0 \leq x \leq \pi \text{ and } y > 0$$

The boundary conditions are

$$u(0,y) = 0 \quad , \qquad u(\pi,y) = 0$$

The initial conditions are

$$\frac{\partial u}{\partial y}(x,0) = 0 \text{ and } u(x,0) = x(\pi - x), \text{ where } 0 \leq x \leq \pi \text{ and } y > 0$$

[50 %]

Question 08

A company is involved in the production of two items (X and Y). The resources need to produce X and Y are twofold, namely machine time for automatic processing and craftsman time for hand finishing. The table below gives the number of minutes required for each item:

Table 1: Required Time for Each Production Steps

Item	Machine Time	Craftsman Time
X	13	20
Y	19	29

The company has 40 hours of machine time available in the next working week but only 35 hours of craftsman time. Both machine and craftsman idle times incur no costs. The revenue received for each item produced (all production is sold) is Rs200 for X and Rs300 for Y. The company has a specific contract to produce 10 items of X per week for a particular customer. Formulate the linear programming model for this optimization problem.

[30 %]

a) Consider the following linear programming problem.

$$\text{Maximize } z = 20x_1 + 30x_2$$

Subject to

$$x_1 + 3x_2 \leq 12$$

$$x_1 + x_2 \leq 6$$

$$3x_1 + 5x_2 \leq 27$$

$$x_1, x_2 \geq 0$$

i. Solve above LP problem by Simplex Method. [45 %]

ii. Verify the solution by graphical method. [25 %]

Question 09

A psychologist was interested in whether different TV shows lead to a more positive outlook on life. A sample of teen-agers were split into 4 groups and then taken to a room to view a program. The four groups saw: The Muppet Show, Action movie, News, No Program. After the program a blood sample was taken, and serotonin levels measured (Remember more serotonin level means more Happy!). The study is done to evaluate that some TV shows make people happier than other.

Table 2: Serotonin Levels in Blood Samples of each Group Members

Muppet Show	Action movie	News	No Program
11	4	4	7
7	8	3	7
8	6	2	5
14	11	2	4
11	9	3	3
10	8	6	4
5			4
			4

a) Identify Dependent variable and Factor variable. [20 %]

b) Write the hypothesis and determine whether there is a significant difference in happiness level among different group of teen-ages at 5% significant level. [70 %]

c) State the assumptions you made. [10 %]

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TABLE

F Distribution: Critical Values of F (5% significance level)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.89	1.85	1.81	1.78
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.86	1.82	1.78	1.75
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.84	1.79	1.75	1.72
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.82	1.77	1.73	1.70
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.80	1.76	1.72	1.69
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.79	1.75	1.71	1.68
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.78	1.73	1.69	1.66
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.76	1.71	1.67	1.64
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.74	1.69	1.66	1.62
250	3.88	3.03	2.64	2.41	2.25	2.13	2.05	1.98	1.92	1.87	1.79	1.73	1.68	1.65	1.61
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.78	1.72	1.68	1.64	1.61
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85	1.78	1.72	1.67	1.63	1.60
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.77	1.71	1.66	1.62	1.59
600	3.86	3.01	2.62	2.39	2.23	2.11	2.02	1.95	1.90	1.85	1.77	1.71	1.66	1.62	1.59
750	3.85	3.01	2.62	2.38	2.23	2.11	2.02	1.95	1.89	1.84	1.77	1.70	1.66	1.62	1.58
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.76	1.70	1.65	1.61	1.58