THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2019/2020)
MHZ3332 / MPZ3132: Engineering Mathematics IB

Date: 29th July 2020 (Wednesday) **Time: 13:30 pm – 16:30 pm**

Instruction:

- **Answer only five questions.**
- **All symbols are in standard notation and state any assumption that you made.**

Important integrals

- $\int e^{ax} sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$
- $\int e^{ax} cosbxdx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$
- $\int f(x)e^{ax}\,dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax}$ …… [Sign alternate $(+-+-+\cdots)$]
- $\int f(x)cosax\,dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax - \cdots$ [Sign alternate in pairs $--++--++--\cdots$)]
- $\int f(x)sinax\,dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax - \cdots$ [Sign alternate in pairs after the first term $(++--++--++--\cdots)$]
- $\int secax\,dx = \frac{1}{a}ln|secax + tanax|$

Q1.

I. Let $f(x)$ be a function defined on the interval $(-1,1)$. Outside of this interval the function is defined by $f(x+2l) = f(x)$.That is $f(x)$ has the period $2l$.The Fourier series of $f(x)$ is defined to be

$$f(x) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right)$$

Write down the formulae for a_0, a_n and b_n. **[15%]**

II. Given the function $f(t)$, where $f(t) = \cos\frac{t}{2}$; $-\pi < t \leq \pi$. **[40%]**

a) Prove that $f(t)$ has the following Fourier series,

$$\frac{2}{\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 - \frac{1}{4}} \cos(nt)$$

b) Find the sum of the following series,

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 - \frac{1}{4}}$$

III. Let $f(x)$ be the function given on the interval $(-2,2)$ by **[45%]**

$$f(x) = \begin{cases} 0, & -2 < x < 0 \\ 2 - x, & 0 < x < 2. \end{cases}$$

Find the Fourier series of $f(x)$.

Q2.

I. Write down the Taylor series expansion of a function $f(x)$ about $x = a$ in usual notation. **[05%]**

II.

a) Find the first four non-zero terms of the Taylor series expansion for the function $f(x) = e^x$ about $x = 0$. **[15%]**

b) Hence obtain the Taylor series expansion for the function $g(x) = e^{-2x^2}$ about $x = 0$. **[10%]**

c) Find the first four non-zero terms of the Taylor series expansion for the function $h(x) = \cos x$ about $x = 0$. **[20%]**

d) Using part (b) and part (c), find the first three non-zero terms of the Taylor series for $l(x) = e^{-2x^2}\cos x$ at $x = 0$. **[20%]**

e) Evaluate the limit, **[10%]**

$$\lim_{x\to 0}\frac{e^{-2x^2}\cos x - 1}{x^2}$$

III. Find the Taylor series for the function $m(x) = x^4 + x - 2$, centered at $x = 1$. **[20%]**

Q3.

I. Solve the following differential equations. **[25%]**

a) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$

b) $\frac{d^2y}{dx^2} - 9y = \sin 2x$

II. Using a suitable trial function, **[40%]**

a) Find a particular integral for the following differential equation,

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = xe^{cx}.$$

b) Find the general solution of the above differential equation.

III. Consider the differential equation $\frac{d^2y}{dx^2} + 4y = 4\cos 2x,$ **[35%]**

a) By using the result, find a particular integral for the above differential equation,

$$\frac{1}{D+\alpha} f(x) = e^{-\alpha x} \frac{1}{D} e^{\alpha x} f(x).$$

You may use $\cos x = \frac{e^{ix} + e^{-ix}}{2}$ and $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$

b) Hence, obtain the general solution of the above differential equation.

Q4.

I. Define the Laplace transformation denoted by $L[f(t)] = F(s)$, of a function $f(t), 0 \leq t < \infty$. **[05%]**

II. Find the Laplace transformation of the following functions. **[30%]**

a) $e^{2x} + 4x^3$

b) $\cos(ax + b)$

c) $e^{2x}(3\cos 6x - 5\sin 6x)$

III. Find the inverse Laplace transformation of the following: **[35%]**

a) $\frac{s^2 - 3s + 4}{s^3}$

b) $\frac{s+2}{s^2 - 4s + 13}$

c) $\frac{4}{(s+2)^4}$

IV. Using the Laplace transformation, solve the following differential equation, given that $y(0) = 0$ and $y'(0) = 1$ **[30%]**

$$\frac{d^2y}{dx^2} + 4\left(y - \frac{dy}{dx}\right) = 3e^{-t}.$$

Q5.

I. A discrete random variable X has the following probability distribution. **[60%]**

$X = x$	0	1	2	3
$P[X = x]$	c	c^2	$c^2 + c$	$3c^2 + 2c$

Find

a) the value of c and hence write down the probability distribution of X.

b) $E(x)$.

c) $V(X)$.

d) $F(x)$, the cumulative probability distribution of X.

II. Fifty plants of a certain variety are planted in 50 pots, numbered from 1 to 50. A pot is selected at random and its number is noted. Let X is the number on the selected pot. **[25%]**

a) What is the probability distribution of X?

b) Find $E(X)$, $V(X)$, and $P[3 \leq X < 5]$.

III. Police use radar unit to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/h and a standard deviation of 10 km/hr. What is the probability that a car stopped at random is travelling at more than 100 km/h? **[15%]**

Q6.

I.

a) Shade the regions satisfying the inequalities $2 \leq |z| \leq 4$ and $\frac{\pi}{6} \leq \arg z \leq \frac{2\pi}{3}$. **[10%]**

b) Express $\text{Log}(\sqrt{3} - i)$ in the form of $x + iy$, where x and y are real numbers. **[15%]**

c) If $f(z) = 2z + 1$, show that f is one to one. Also find the inverse function of f. **[15%]**

II. The curve is defined as $x = t,\ y = t^2,\ z = t^3$. Find the following vectors at $t = 1$: **[60%]**

a) tangent vector.

b) unit tangent vector.

c) normal vector.

d) unit normal vector.

Q7.

I. Find the moment of inertia of the following bodies about the given axis. **[50%]**

a) A uniform circular disc of radius r, the axis perpendicular to the plane of the disc passing through the center.

b) A uniform solid cone of radius r and height h about the axis of the cone.

c) A uniform rod AB of mass m and length $2a$, the axis perpendicular to the rod passing through the midpoint of the rod.

II. A uniform circular plate of radius a has a mass M. A hole of radius $a/5$ is punched in the plate. The center of the hole being a distance of $3a/5$ from the center of the plate. Find the moment of inertia of the plate about, **[50%]**

a) The axis through the center of the hole perpendicular to the plate.

b) A line through the center of the disc perpendicular to its plane.

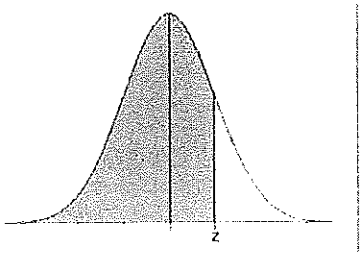
Q8.

I. A semi-circular lamina of radius r and center O immersed in a homogeneous liquid with its plane vertical and the straight edge on the surface. Show that the center of the pressure lamina is at a depth $\frac{3\pi r}{16}$ from O. **[60%]**

II. A rectangle is immersed vertically in water with two sides horizontal and at depth a and $a + b$ respectively below the water surface. Prove that the distance of the center of pressure from the upper side is $\frac{b(3a + 2b)}{3(2a + b)}$. **[40%]**

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Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998