The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Study Programme	: Bachelor of Technology Honors in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: MHZ3531/MPZ3231: Engineering Mathematics IA**
Academic Year	: 2019/2020
Date	: 29th September 2020
Time	: 0930-1230hrs
Duration	: **3 hours**

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Four (4)** pages.
3. Answer any **Five** out of **Eight** questions.
4. Show all steps clearly
5. Answer for each question should commence from a new page.
6. This is a Closed Book Test **(CBT).**
7. **Programmable** calculators are not allowed.
8. Do not use red color pen.

Q1 (a) i) Using the principle of mathematical induction, $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^n = \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$, (10%)

where $n \in Z^+$.

ii) Prove that the roots of the equation $|A - \lambda I| = 0$ are -1 and 6, where I (10%)

is the 2×2 unit matrix and $A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$.

α) Taking $\lambda_1 = -1$ and $\lambda_2 = 6$. if $AX_1 = \lambda_1 X_1$ and $AX_2 = \lambda_2 X_2$, (15%)

Prove that $X_1 = t\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $X_2 = r\begin{bmatrix} 2 \\ 5 \end{bmatrix}$,where t, r are parameters.

β) Prove that $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ are linearly independent. (10%)

iii) Let $U = \begin{bmatrix} 1 & 2 \\ -1 & 5 \end{bmatrix}$ and $D = \begin{bmatrix} -1 & 0 \\ 0 & 6 \end{bmatrix}$.

α) Show that $A = UDU^{-1}$. (10%)

β) Hence find A^n, where $n \in Z^+$. (15%)

(b) Using row and column operations, find all the factors of $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ac \\ (a+b)^2 & c^2 & ab \end{vmatrix}$. (30%)

Q2. (a). Let $f(x) = \dfrac{x^3}{x^2-3}$, where $x \neq \pm 1$.

i). Find the vertical and slant asymptotes of the curve of $y = f(x)$. (20%)

ii). Prove that $\dfrac{dy}{dx} = \dfrac{x^2(x-3)(x+3)}{(x-\sqrt{3})^2(\sqrt{3}+x)^2}$. (05%)

iii). Determine the sign of $\dfrac{dy}{dx}$ as x varies from $-\infty$ to ∞. (10%)

iv). Find the turning points of the curve of $y = f(x)$. (10%)

v). Sketch the graph of $y = f(x)$ and hence deduce the graph of $y = |f(x)|$. (25%)

(b). If $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$ prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$. (30%)

Q3 (a) If, $y \in C$, prove that $|x-y| < |1-\bar{x}y|$ if and only if $\{|x| < 1$ and $|y| < 1\}$ or $\{|x| > 1$ and $|y| > 1\}$. (25%)

(b) i) State De Moivre's theorem. (10%)

ii) If $z = \cos\theta + i\sin\theta$, then prove that $z^n + \dfrac{1}{z^n} = 2\cos n\theta$ and $z^n - \dfrac{1}{z^n} = 2i\sin n\theta$. (15%)

iii) Hence prove that $8\sin^4\theta = \cos 4\theta - 4\cos 2\theta + 3$ and $8\cos^4\theta = \cos 4\theta + 4\cos 2\theta + 3$. (30%)

(c) Find the graph of $Re(z+5) = |z-5|$. (20%)

Q4 (a) Prove that the normal line to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is $y + tx = 2at + at^3$. (15%)

i) Prove that generally three normal lines can be drawn from a point P to the parabola $y^2 = 4ax$. (25%)

ii) If two of those normal lines are perpendicular to each other, then prove that the point P lies on the parabola $y^2 = a(x-3a)$. (35%)

(b) Find the coordinates of the two points on the line $\dfrac{x+1}{2} = \dfrac{y+3}{2} = \dfrac{z-4}{-1}$ such that the perpendicular distance from each of those two points on that the line to the plane $x + 2y - 2z = 9$ is 8 units. (25%)

Q5 (a) Consider the differential equation
$[3y^2 + (k+1)x^2y]dy - [(k-3)xy^2 - 3x^2]dx = 0$, where $k \in \mathbb{R}$.

i) Find the value of k such that the above differential is exact. (20%)

ii) For the above value of k solve the differential equation. (20%)

(b) Solve the differential equation $\cos x \frac{dy}{dx} + y \sin x = 2020x^{2019} \cos^2 x$ (30%)

(c) Solve the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$. with the boundary conditions $y(0) = 0$ and $y'(1) = 6$. (30%)

Q6 Define the vector product of two vectors.

(a) Vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are such that $\boldsymbol{a} \times \boldsymbol{b} = 3\boldsymbol{a} \times \boldsymbol{c}$. (50%)

Prove that $\boldsymbol{a} = \boldsymbol{0}$ or $\boldsymbol{b} = 3\boldsymbol{c} + k\boldsymbol{a}$ where $k \in R$.

If $|\boldsymbol{a}| = 5$, $|\boldsymbol{b}| = 3$, $|\boldsymbol{c}| = \sqrt{3}$ and $\boldsymbol{b} \cdot c = 0$, then find the value of k.

For the above value of k find the angle between

i) $\boldsymbol{a}$ and $\boldsymbol{b}$. ii) $\boldsymbol{a}$ and $\boldsymbol{c}$.

(b) $\vec{OA} = 2\mu\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, $\vec{OB} = \mu\mathbf{i} + (\mu - 1)\mathbf{j} + \mathbf{k}$ and $\vec{OC} = 4\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ (50%)

Find the area of the triangle OAB and volume of the parallelepiped with the edges OA, OB, OC in terms of μ.

Deduce the values of μ such that O, A, B, C are coplanar.

Q7 (a) Let $f(x) = x^3 - 3x - 9$

i) Sketch the graph of $y = f(x)$. (10%)

ii) Show that $x^3 - 3x - 9 = 0$ has a root in the interval $(2, 3)$. (10%)

iii) Using the method of bisection, find the value of the root to correct nine decimal places. (30%)

iv) Find the number of iterations such that the error is less than to 10^{-4}. (15%)

(b) Let $f(x) = x^2 - \sqrt{N}$.

i) Using Newton Raphson formula, prove that $x_{k+1} = \frac{1}{2}\left(x_k + \frac{N}{x_k}\right)$. (10%)

ii) Hence, find the value $\sqrt{6}$ of correct to four decimal places. (25%)

Q8 (a) State the Baye's theorem. (10%)

$U_1, U_2, U_3 \ldots\ldots\ldots\ldots U_{13}$ are identical urns and each contains 13 balls that are only different in colour. U_r contains r number of red balls, where $r = 1, 2, 3, \ldots\ldots 13$. One ball is selected at random from an urn which is selected at random. If the colour of the selected ball is red, then find the probability that the selected urn is U_9. (40%)

(b) Define the mean and variance of the continuous random variable. (10%)

The probability density function of a continuous random variable X is

$$f(x) = \begin{cases} \lambda x(3-x)(3+x) & 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

i) Find the value of constant λ. (10%)

ii) Calculate the expectation, variance, and mode of X (15%)

iii) Prove that the median of X is $1 \cdot 62$. (15%)

End

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