The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Study Programme	: Bachelor of Technology Honors in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: MHZ3551: Engineering Mathematics I**
Academic Year	: 2019/2020
Date	: 29th September 2020
Time	: 0930-1230hrs
Duration	: **3 hours**

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Four (4)** pages.
3. Answer any **Five (5)** out of **Eight (8)** questions.
4. Show all steps clearly
5. Answer for each question should commence from a new page.
6. This is a Closed Book Test **(CBT).**
7. **Programmable** calculators are not allowed.
8. Do not use red color pen.

Q1. (a) Let p and q be any two propositions. By using laws in propositional logic, show that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology. (25%)

(b) A barber is a person who shaves all those do not shave themselves. If $B(x)$: x is a barber and $S(x, y)$: x shaves y, then translate the above definition as a quantification formula. Show that this definition leads to a contradiction. (25%)

(c) Let a be an integer. Use a proof by cases, to show that, if a is not divisible by 3, then $a^2 - 1$ is divisible by 3. (25%)

(d) Use a method of indirect proof to prove that if $3n + 2$ is an even integer for the integer n then, n is even. (25%)

Q2 (a) Let A and B be any two subsets of the universal set U. Using the laws of sets prove that $A \cap B = B - (B - A)$. (20%)

(b) Show that the relation σ defined on $\mathbb{N}$ by $\sigma = \{(x, y) | y = nx, n \in \mathbb{Z}^+\}$ is a partially ordered relation. (20%)

(c) Let $f: x \to x^2 - 4$ for $x \in (-\infty, 0)$. Show that f is a one to one function and find the inverse function of f. (30%)

(d) Let $f: x \to x^2$ for $x \in (0, 2)$ and $g: x \to 2x + 1$ for $x \in (0, 1)$. Find R_g, R_f and the function $f \circ g$. (30%)

Q3 (a) Let $P = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ -1 & 2 & -2 \\ 2 & 2 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

i) Prove that $P^T P = I$ and $D = P^T A P$. (30 %)

ii) Hence, find A^n where $n \in Z^+$. (35 %)

(b) Applying elementary row or column operations find the factors of $\begin{vmatrix} 1 & a^2 & a^4 \\ 1 & b^2 & b^4 \\ 1 & c^2 & c^4 \end{vmatrix}$. (35 %)

Q4 (a) Let $(a_n)_{n \in Z^+}$ be a sequence of positive numbers such that $a_1 > \sqrt{5}$ and

$$a_{n+1} = \frac{5(1 + a_n)}{5 + a_n} \text{ for } n \in \mathbb{Z}^+.$$

i) Express $(a_{n+1})^2 - 5$ in terms of a_n. (25%)

ii) Using the principle of mathematical induction prove that $a_n > \sqrt{5}$.

iii) Deduce that $(a_n)_{n \in Z^+}$ is a strictly decreasing sequence and find $\lim_{n \to \infty} a_n$. (15%)

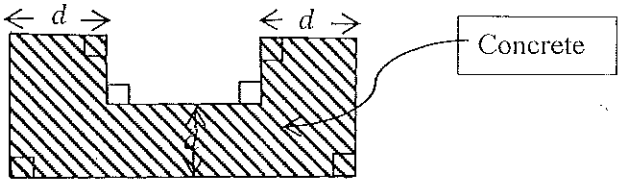
(b) Use D' Alembert's ratio test, to show that $\sum_{n=1}^{\infty} \frac{2^n + 2020}{3^n}$ is convergent. (20%)

(c) Show that $\lim_{(x,y) \to (0,0)} \frac{2x^2 - 5y^2}{x^2 + 3y^2}$ does not exist. (20%)

(d) Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ if $f(x, y) = e^x \sin y$. (20%)

Q5 (a) Evaluate $\lim_{x\to 0}\left(\frac{e^{2sinx}-1}{\sqrt{3x+4}-\sqrt{2x+3}}\right)$. (30%)

(b) The cross section of a uniform concrete drain that is to be made is shown in the following figure. The area of the inner cross section of the drain is $2a^2$ square meters and the width of the concrete is d meters. The sides are vertical, and the base is horizontal. Using the test of second derivative, find the inner dimensions of the drain with the minimal concrete. (35%)



(c) Let the minimal radius of a car tire tube is R meters and the radius of its cross section is r meters. Use integration to find the volume of the car tire tube. (35%)

Q6 (a) Using an integrating factor, solve the differential equation (50%)

$$x\frac{dy}{dx}-y=\frac{x^2}{\sqrt{-x^2-6x-5}}.$$

Substituting $z=y^2$, deduce the general solution of the differential equation

$$2xy\frac{dy}{dx}=y^2+\frac{x^2}{\sqrt{-x^2-6x-5}}.$$

(b). Show that the differential equation $y(1+xy)dx+x(1-xy)dy=0$ is not exact. If it has an integrating factor of the form x^ny^m then, find the values of m,n. Hence, find the general solution of the differential equation. (50%)

Q7. (a). Consider the differential equation, $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=0$.

i) Find the general solution of the above differential equation. (20%)

ii) Determine the solution if $y(0)=\left(\frac{dy}{dx}\right)_{x=0}=4$. (15%)

iii) The differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=20cosx-19sinx$ has a trial function of the form $y_T=Acosx+Bsinx$.

(α) Find the values of A and B. (20%)

(β) Find the general solution of the above differential equation. (15%)

(b) Let $y_P = ke^x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 36e^x$$

i) Find the value of k. (15%)

ii) Find the general solution of the above differential equation. (15%)

Q8 A particle is projected vertically upwards with speed u from a point O. The air resistance on a unit mass is $\frac{v^4}{u^4}g$, where v is the velocity of the particle at a distance x from O.

(a) Prove that $\frac{d}{dx}(v^2) = -2g\left(\frac{u^4 + v^4}{u^4}\right)$. (35%)

(b) Deduce that the maximum height attained by the particle is $\frac{\pi u^2}{8g}$. (35%)

(c) Find the speed of the particle when it returns to O. (30%)

End

-Copyright reserved -