THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2019/2020)
MPZ3552: Engineering Mathematics II

Date: 29th July 2020 (Wednesday) **Time: 13:30 – 16:30**

Instruction:

- **Please answer a total of five (05) questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

Important integrals

- $\int e^{ax} sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$
- $\int e^{ax} cosbxdx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$
- $\int f(x)e^{ax}\,dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax}$ …… [Sign alternate $(+ - + - + \cdots)$]
- $\int f(x)cosax\,dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax - \cdots$ [Sign alternate in pairs $- - + + - - + + - - \cdots$)]
- $\int f(x)sinax\,dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax - \cdots$ [Sign alternate in pairs after the first term $(+ + - - + + - - + + - - \cdots)$]
- $\int secax\,dx = \frac{1}{a}ln|secax + tanax|$

SECTION – A

Q1.

I. Saman measured the height, (in **cm**) of each of the chili plants in his green house. Here are his results. **[45%]**

22	20	32	35	23	34
28	22	42	29	22	34
50	45	38	37	28	46
20	40	22	42	47	22
50	42	24	39	33	22

a) Construct a frequency table for above results by identifying appropriate class size.

b) Present the data in the form of a histogram.

c) Draw the corresponding ogive.

II. Nimal had been working on programing and updating a web site for his company for the last year. The following numbers represent the number of hours Nimal has worked on this web site for the past 10 months. **[55%]**

20	25	22	32	35
23	28	30	35	35

a) Find the mean, mode, median number of hours that Nimal worked on this web site.

b) What can you say about the shape of the distribution of the working hours?

c) Find range, standard deviation, inter quartile range of the working hours.

d) Draw the box-whisker plot for this data.

Q2.

I. A discrete random variable X has the following probability distribution. **[60%]**

$X = x$	0	1	2	3
$P[X = x]$	c	c^2	$c^2 + c$	$3c^2 + 2c$

Find,

a) the value of c and hence write down the probability distribution of X.

b) $E(X)$.

c) $V(X)$.

d) $F(x)$, the cumulative probability distribution of X.

II. Fifty plants of a certain variety are planted in 50 pots, numbered from 1 to 50. A pot is selected at random and its number is noted. Let X is the number on the selected pot. **[15%]**

a) What is the probability distribution of X?

b) Find $E(X)$, $V(X)$ and $P[3 \leq X < 5]$.

III. Police use radar unit to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/h and a standard deviation of 10 km/hr. What is the probability that a car stopped at random is travelling at more than 100 km/h? **[25%]**

SECTION – B

Q3.

I. Find the direction cosines the direction $\boldsymbol{AB}$. If the position vectors of the point A and $\boldsymbol{B}$ are $4\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ and $8\boldsymbol{i} + 2\boldsymbol{j} + 6\boldsymbol{k}$. **[10%]**

II. Let vector $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are $2\boldsymbol{i} + 3\boldsymbol{j} + 6\boldsymbol{k}$, $3\boldsymbol{i} - 6\boldsymbol{j} + 2\boldsymbol{k}$, and $6\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$ respectively.

a) Find $\boldsymbol{a} \times \boldsymbol{b}$. **[05%]**

b) Deduce that $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ are mutually perpendicular vectors. **[10%]**

III. Show that the equation of the line passing through given two points with position vector $\boldsymbol{a}$ and $\boldsymbol{b}$ is $\boldsymbol{a} + \lambda(\boldsymbol{b} - \boldsymbol{a})$, where λ is a parameter.

Find the cartesian equation of the line of $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$.

[25%]

IV. A particle P moves with velocity $2\boldsymbol{i} - 3\boldsymbol{j}$ from (4, 5). At the same instant a particle Q, moving in the same plane with velocity $4\boldsymbol{i} + \boldsymbol{j}$, passes through point (0, -3). Find whether P and Q collide or not. **[20%]**

V. A particle A starts to move with constant velocity $5\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ from point $(1, 0, 2)$. Another particle B starts to move at another time, moving in the same plane with velocity $2\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$, passes through a point $(2,7,6)$. Find whether A and B collide. Neglect the air resistance and the masses of the particles. **[30%]**

Q4.

I.

a) Shade the regions satisfying the inequalities **[10%]**
$2 \leq |z| \leq 4$ and $^{\pi}/_{3} \leq arg\, z \leq {}^{4\pi}/_{3}$

b) Express $\text{Log}(\sqrt{3} - i)$ in the form of $x + iy$, where x and y are real numbers. **[15%]**

c) If $f(z) = 2z + 1$, show that f is one to one. Also find the inverse function of f. **[15%]**

II. Find the value of the complex number $(\sqrt{3} - i)^6$. **[15%]**

III. Using the De Moivre's theorem prove that **[45%]**

$\cos 6\theta = 32cos^6\theta - 48cos^4\theta + 18cos^2\theta - 1.$

Deduce that

$sin6\theta = sin2\theta\ (3 - 16\ sin^2\theta + 16\ sin^4\theta).$

SECTION – C

Q5.

I. Let the function $f(x) = x^3 + 3x - 5$,

a). Identify a suitable interval to apply the bisection method to find the root of the function $f(x)$. **[10%]**

b). Find the root of the above function using the method of bisection. **[30%]**

II. Find the solution of the equation $x^3 = 20$ up to 3 decimal places using Newton - Raphson method. You may use the initial guess as $x_0 = 3$. **[30%]**

III. Solve the following system with Gaussian elimination of the variables with partial pivoting. **[30%]**

$$x - 2y - z = 0$$
$$x - y + 2z = -9$$
$$2x - y - 11z = -43\,.$$

Q6.

I. Construct a polynomial for the values given in following table, using Lagrange's interpolation formula and estimate $f(2)$. **[40%]**

x	-2	-1	0	4
y	-2	4	1	8

II. The value of the function $y = f(x)$ at some discrete values are given in following table. Use Newton's forward difference interpolation formula to evaluate *f (3).* **[25%]**

x	0	5	10	15	20
f(X)	10	16	38	82	154

III.

a). Evaluate $\int_0^1 (4x - 3x^2)$ by using Trapezoidal and Simpson's rules. Use 10 subintervals for the calculation. **[30%]**

b). Compare the result of Trapezoidal rule and Simpson's rule with the actual value. **[05%]**

SECTION – D

Q7.

I. Define the McLaurin's series expansion and let $f(x) = \ln(1-x)$ prove that

$$ln(1-x) + \sum_{r=0}^{\infty} \frac{x^{r+1}}{r+1} = 0 \text{ where } |x| < 1.$$

Using the above formula prove that $\frac{1}{2} ln\left(\frac{7+x}{7-x}\right) = \sum_{r=0}^{\infty} \frac{x^{2r+1}}{(2r+1)} \left(\frac{1}{7}\right)^{2r+1}$.

[50%]

II. Define the Laplace transform for a function $f(t), 0 \leq t < \infty$.
Find the Laplace transformation of the following functions. **[05%]**

a) Find the Laplace transformation of $e^{2x} + 4x^3$ and
$e^{2x}(3\cos 6x - 5\sin 6x)$. **[25%]**

b) Find the inverse Laplace transformation of the following: **[20%]**

$$\frac{s^2 - 3s + 4}{s^3}$$

Q8.

I. Let $f(x)$ be a periodic that is Reimann integral function on the interval $(-l, l)$.
Fourier series of $f(x)$ is defined as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right)$$

write down the formulae for a_0, a_n and b_n. **[15%]**

II. Given the function $f(t) = cos\frac{t}{2}$, where; $-\pi < t \leq \pi$. **[40%]**

a) Prove that the f has the Fourier series,

$$\frac{2}{\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 - \frac{1}{4}} \cos(nt)$$

b) Find the sum of the series,

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 - \frac{1}{4}}$$

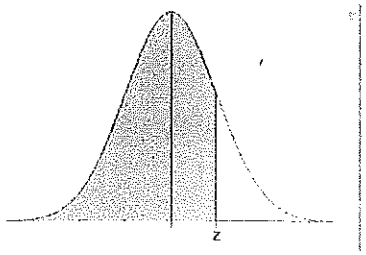
III. Let $f(x)$ be a function defined on the interval $(-2,2)$ by

$$f(x) = \begin{cases} 0, & -2 < x < 0 \\ 2 - x, & 0 < x < 2. \end{cases}$$

Find the Fourier series of $f(x)$. **[45%]**

--END--

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998