The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	:	Bachelor of Technology (Engineering)
Name of the Examination	:	Final Examination
Course Code and Title	:	**MEX6278 / MEX5230 Fluid Mechanics**
Academic Year	:	2014/15
Date	:	September 16, 2015
Time	:	0930 hrs. – 1230 hrs.
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer any 5 questions only.

Question 1

1.1 Derive that, the pressure (P_1) at a point in the troposphere, at which the temperature is T_1, can be expressed as,

$$\frac{P_1}{P_0} = \left[\frac{T_1}{T_0}\right]^{\frac{g}{mR}}$$

and also show that, the pressure (P_2) at a point situated at an altitude of Z_2 in the stratosphere can be derived as,

$$\frac{P_2}{P_*} = e^{\left[-\frac{g}{RT_*}(Z_2 - Z_*)\right]}$$

where, P_o and T_o are pressure and temperature at sea level, P_* and T_* are pressure and temperature at an altitude Z_* (Z_*=11000m) and g is gravitational acceleration, R universal gas constant and 'm' temperature laps rate.

1.2 At the top of a mountain the temperature is -50C and the mercury barometer reads 566mm, whereas the reading at the foot of mountain is 749mm. Assuming that atmospheric temperature decreases with increasing altitude at a uniform rate (temperature laps rate) of 0.0065 K/m, calculate the height of the mountain. Take the gas constant for air as (R) 287J/kgK and neglect thermal expansion of mercury.

Question 2

2.1 Derive an expression for the depth of paraboloid formed by the surface of liquid contained in a cylindrical tank which rotated at a constant angular velocity ω about its axis.

2.2 A hemispherical projection of diameter 1.5m exists on one of the vertical sides of a tank as shown in figure *Q2*. If the tank contains water (specific weight 10kN/m^3) to an elevation of 3.5m above the center of hemisphere, determine the horizontal and vertical forces acting on the projection.

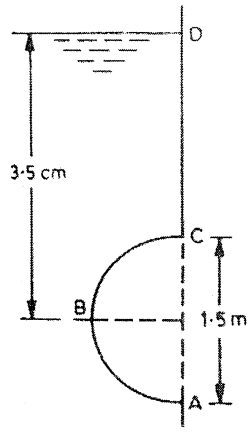


Figure Q2

Question 3

3.1 Two reservoirs with a surface level difference of 20m are to be connected by a 1m diameter 6km long pipe. What will be the discharge when cast iron pipe of average roughness (k) 0.3mm is used?

3.2 What will be the percentage increase in the discharge if the cast iron pipe is replaced by a steel pipe of average roughness (k) 0.1mm? Neglect all local losses.

The Karman-Prandtl equation for friction factor for turbulent flow through rough pipes is given by

$$\frac{1}{\sqrt{f}} = 2log\left(\frac{R_0}{k}\right) + 1.74$$

where f = friction factor, R_0 = pipe radius, k = average roughness

Question 4

4.1 State the Buckingham's Pi theorem used in dimension analysis.

4.2 By using the Buckingham's Pi theorem, show that the power (P) developed in a water turbine can expresses as :

$$P = \rho N^3 D^5 \emptyset \left[\frac{D}{B}, \frac{\rho D^2}{\mu}, \frac{ND}{\sqrt{gH}}\right]$$

where D and B are the diameter and width of the runner, N is the speed in revolutions per minute, H is the operating head, μ and ρ are respectively the coefficient of dynamic viscosity and mass density of the liquid.

Question 5

5.1 Two source each of strength m are placed at the points (-a,0), (a,0) and a sink of strength 2m at the origin. Show that the stream function is given by

$$(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$$

where λ is a variable parameter for a given stream line.

Show that the speed at any point is

$$\frac{2ma^2}{r_1 r_2 r_3}$$

where r_1, r_2 and r_3 are the distances of the point from the two sources and the sink respectively.

You may use the following relationships

$$z = (x + iy) \text{ and } \ln(\alpha + i\beta) = \frac{1}{2}\ln(\alpha^2 + \beta^2) + i\, tan^{-1}(\beta/\alpha)$$

Question 6

6.1 Consider an irrotational flow, past a fixed circular cylinder of radius *'a'* . The flow has a uniform velocity U at infinity. The cylinder is of infinite length and is placed with its axis perpendicular to the direction of flow. Assuming that the flow past the cylinder is inviscid and incompressible, the above flow can be described by the complex potential,

$$W(z) = U(z + a^2/z).$$

a) Determine the pressure on the surface of the cylinder using the Bernoulli's theorem, in which the contribution due to the gravity can be neglected.

b) Use Blasius theory, to show that the net force acting upon the cylinder surface is zero.

c) If the modified complex potential is

$$W(z) = U(z + a^2/z) + (i\Gamma/2\pi)\ln(z/a)$$

Where Γ is the circulation (or vortex strength) in the clockwise direction around the cylinder.

i) Show that the stream function describing the flow is

$$U_r(1 - a^2/r^2)sin\theta + (\Gamma/2\pi)ln(r/a)$$

ii) Show that the velocity components are

$$U_r = U(1 - a^2/r^2)cos\theta$$

and

$$U_\theta = -U(1 + a^2/r^2)sin\theta - (\Gamma/2\pi r)$$

Question 7

7.1 Explain the significance of the boundary layer displacement thickness δ^* and the momentum thickness θ and express them as integrals of the boundary layer velocity profiles on a smooth flat plate.

7.2 Calculate the ratio of displacement thickness to disturbance thickness *(δ^*/δ)* for a laminar boundary layer with a velocity profile given by

$$\frac{u}{u_0} = 2\frac{y}{\delta} - \frac{y^2}{\delta^2}$$

7.3 Calculate the ratio θ/δ for a turbulent boundary layer when velocity profile is given by

$$\frac{u}{u_0} = \left(\frac{y}{\delta}\right)^{1/7}$$

Question 8

8.1 Making appropriate assumptions to the Navier-Stokes equation in cylindrical coordinates, show that the steady flow in a circular pipe could be described by the simplified mathematical model

$$r\frac{dp}{dx} = \mu\frac{d}{dr}\left(r\frac{du}{dr}\right)$$

Also derive expressions for u and V_r.

where r and x are the space co-ordinates, u is the axial velocity, $\frac{dp}{dx}$ is the pressure gradient and μ is the dynamic viscosity.

Note:

- *Navier–Stokes equations in cylindrical co-ordinates for incompressible flow, in usual notation are given below.*

NS Equation for x direction - Axial flow

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+V_r\frac{\partial u}{\partial r}+\frac{V_\theta}{r}\frac{\partial u}{\partial \theta}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

NS Equation for r direction - Radial flow

$$\rho\left(\frac{\partial V_r}{\partial t}+u\frac{\partial V_r}{\partial x}+V_r\frac{\partial V_r}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}-\frac{V_\theta^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{\partial^2 V_r}{\partial x^2}+\frac{\partial^2 V_r}{\partial r^2}+\frac{1}{r}\frac{\partial V_r}{\partial r}-\frac{V_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right)$$

NS Equation for θ direction - Circumferential flow

$$\rho\left(\frac{\partial V_\theta}{\partial t}+u\frac{\partial V_\theta}{\partial x}+V_r\frac{\partial V_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{V_rV_\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left(\frac{\partial^2 V_\theta}{\partial x^2}+\frac{\partial^2 V_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial V_\theta}{\partial r}-\frac{V_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial V_r}{\partial \theta}\right)$$

- *Continuing equation for incompressible fluid flow in terms of cylindrical co-ordinates is,*

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta)+\frac{\partial}{\partial u}(u)=0$$

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