THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2019/2020)
MHZ4553: Engineering Mathematics III

Date: 12^{th} August 2020 Time: 09:30 a.m.-12:30 p.m..

Instructions:

- This paper consists of NINE (9) questions in FIVE (5) pages.
- Relevant Statistic Tables and equations are provided.
- Answer any FIVE (5) questions ONLY. All questions carry equal marks.
- Answer for each question should commence from a new page.
- State any assumptions you required.
- This is a closed book test.
- Show all your workings.
- All symbols are in standard notation.
- Do not use red color pen.

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1. (a) Let $f(x,y) = e^x \sin y$.

   i. Find the quadratic approximation $Q(x,y)$, for f at the origin. *(marks 20)*

   ii. Find a linear function $L(x,y)$ and a symmetric matrix A such that

   $$Q(x,y) = L(x,y) + \frac{1}{2}(x,y)A\begin{pmatrix} x \\ y \end{pmatrix}.$$ *(marks 15)*

(b) Let $A = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}$.

i. Find an orthogonal matrix P such that diagonalize A. (*marks 30*)

ii. Find a change of variables that reduce $(x, y)A \begin{pmatrix} x \\ y \end{pmatrix}$ to a sum of squares and express $(x, y)A \begin{pmatrix} x \\ y \end{pmatrix}$ in terms of the new variables. (*marks 35*)

2. (a) Let $g(x, y) = xy + x + y$. Find the gradient of g and the rate of change of g at $(1, 0)$ in the direction of $\bar{u} = (2, 1)$. (*marks 30*)

(b) Let $f(x, y) = x^2 + y^2 + z^2$.

i. Find global extrema of f and explain the nature of the extrema. (*marks 25*)

ii. How your answer of (i) will change if you have the constraint $x + y + z = 1$? What can you say about your new answer, whether it is a maximum, minimum or a saddle point? (*marks 45*)

3. (a) Let $f(z) = u(r, \theta) + iv(r, \theta)$, where $z = re^{i\theta}$. State Cauchy-Riemann equations in terms of polar coordinates. (*marks 20*)

(b) Find an analytic function f in domain D^* such that $Im(f) = Arg(z)$ and $f(1) = 0$. Give your solution in terms of z. (*marks 65*)

(c) What is the domain D^* of f? (*marks 15*)

4. (a) Evaluate $\int_c f.dr$, where $f(x, y) = (x + y^2)i + y^2 j$ and c is the upper half circle with center $(0, 0)$ and radius 2 in positive direction. (*marks 40*)

(b) Evaluate $\int_\gamma z^2 dz$, where γ: straight line from $(1, 1)$ to $(2, 3)$. (*marks 25*)

(c) Find $\int_\Gamma \frac{2z^2 + z}{z^2 - 1}$, where $\Gamma : |z - 1| = 1$ and positively oriented. (*marks 35*)

5. (a) Find the series solution of $y'' + y = 0$. *(marks 45)*

(b) Solve the following boundary value problem using the variable separable method.

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{2}\frac{\partial u}{\partial t} \quad 0 < x < 2, \quad 0 < t$$

$$u(0,t) = u(2,t) = 0$$

$$u(x,0) = 3\sin 2\pi x.$$

(marks 55)

6. (a) Consider the initial value problem

$$y' = 2xy \quad \text{with } y(0) = 1. \tag{1}$$

i. Find the exact solution of (1). *(marks 10)*

ii. Use the **Euler** method with step size $h = 0.1$ to find an approximate value of the solution of (2) at $x = 0.2$. *(marks 30)*

(b) Consider the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{2}\frac{\partial u}{\partial t} \tag{2}$$

with

$$u(x,2) = 40 \text{ for } 0 < x < 2$$

$$u(0,t) = 40 \text{ and } u(2,t) = 0 \text{ for } 0 \le t < 2$$

which defined in the rectangular;

$$R = \{(x,t) \in \mathbb{R}^2 \ : \ 0 < x < 2, \ 0 \le t < 2\}.$$

Use finite difference method with step size $h = k = 1$, to find an approximate value to solution of (2) at the point $(1,0)$, i.e., find $u(1,0)$.

Note that:

$$\frac{\partial u}{\partial x} = \frac{u_{i+1,j} - u_{i,j}}{h}$$

$$\frac{\partial u}{\partial t} = \frac{u_{i,j+1} - u_{i,j}}{k}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{h^2}(u_{i+1,j} + u_{i-1,j} - 2u_{i,j}).$$

(marks 60)

7. Let U, V be subspaces of $\mathbb{R}^2$. Let $T : U \to V$ given by $T(x, y) = (2x + y, x - 3y)$.

   (a) Show that T is a linear transformation. *(marks 25)*

   (b) Show that $\mathcal{B} = \{v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\}$ is a basis for the range of T. *(marks 40)*

   (c) Determine the matrix of T with respect to the standard basis $\{e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\}$ of U and the basis $\mathcal{B} = \{v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\}$ of range of T. *(marks 35)*

8. (a) A machine for making precision cuts in dimension lumber produces studs with lengths that vary with standard deviation 3 inch. Ten trial cuts are made to check the machine's calibration. The mean length of the studs produced is 104.8 inches.

   i. Determine the standard error for mean length of the studs. *(marks 10)*

   ii. Find an interval estimate for the population mean at 5% significant level. State the assumptions that you have made for this calculation. *(marks 20)*

   iii. Interpret your answer in part 8(a)ii. *(marks 10)*

   (b) Calcium is a vital nutrient for healthy bones and teeth. The National Institutes of Health (NIH) recommends a calcium intake of 1300 milligrams (mg) per day for teenagers. The NIH is concerned that teenagers aren't getting enough calcium, on average. Researchers decide to perform a study to estimate daily calcium intake in the population of teenagers. They ask a random sample of 40 teens to record their food and drink consumption for 1 day. The researchers then compute the calcium intake for each student. Data analysis reveals that $\bar{x} = 1198mg$ and $s_x = 411mg$.

   i. State the hypothesis for decision makers claim. *(marks 10)*

   ii. Test the above hypothesis using an appropriate method at 5% significant level. *(marks 30)*

   iii. Interpret your answer in part 8(b)ii. *(marks 10)*

   iv. State the assumptions that you have made. *(marks 10)*

9. The blood pressure $pmmHg$ and the age t years of 10 Covid-19 patients are shown in the table 1 below.

Table 1: The Blood Pressure of Covid-19 Patients.

Patient	A	B	C	D	E	F	G	H	I	J
t	40	67	48	35	56	26	60	43	65	42
p	95	158	120	88	150	80	155	102	163	98

(a) Identify the independent variable and the dependent variable. (*marks 10*)

(b) Calculate the correlation coefficient for above data. (*marks 25*)

(c) Interpret the correlation coefficient. (*marks 10*)

(d) Draw the scatter diagram for the dependent variable against the independent variable. (*marks 10*)

(e) Find the equation for best fitted regression line of p on t. (*marks 25*)

(f) Draw the best fitted line on the scatter plot in part (2) (d). (*marks 10*)

(g) Use your regression line to estimate the blood pressure of a 40 years old Covid-19 patient. (*marks 10*)

End.

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Notes

Correlation coefficient Value

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

Parameters of the linear regression model

$$\widehat{\beta_1} = \frac{\sum_{i=1}^{n}(x_i y_i) - \frac{(\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}}{\sum_{i=1}^{n} {x_i}^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}$$

$$\widehat{\beta_0} = \bar{y} - \widehat{\beta_1}\bar{x}$$

Best fitted line

$$\hat{y} = \widehat{\beta_0} - \widehat{\beta_1}x$$

Probabilities for the standard normal distribution

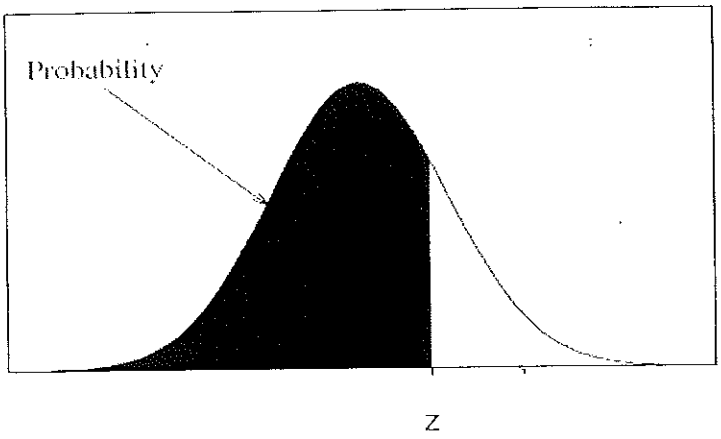
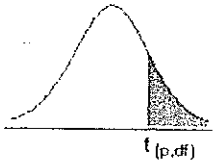


Table entry for z is the probability lying to the left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

t table with right tail probabilities



df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.4	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579		<	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548			35539	5.0413
9	0.260955	0.702722	1.383029	1.8331				4.7809
10	0.260185	0.699812	1.372184	1.8				769
11	0.259556	0.697445	1.363430	1				
12	0.259033	0.695483	1.356217					
13	0.258591	0.693829	1.35017					208
14	0.258213	0.692417	1.345					4.1405
15	0.257885	0.691197	1.340606					4.0728
16	0.257599	0.690132	1.336757	1.74			3	4.0150
17	0.257347	0.689195	1.333379	1.739607			823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.100	2	87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905