THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2020/2021)

MHZ4553: Engineering Mathematics III

Date: 23rd February 2022 **Time: 09:30 a.m.-12:30 p.m..**

Instructions:

- This paper consists of NINE (9) questions in SIX (6) pages.
- Answer any FIVE (5) questions ONLY. All questions carry equal marks.
- Answer for each question should commence from a new page.
- Relevant Statistic Tables and equations are provided.
- State any assumptions you required.
- This is a closed book test.
- Show all your workings.
- All symbols are in standard notation.
- Do not use red color pen.

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1. (a) i. Find a symmetric matrix A such that $(x,y)A\begin{pmatrix}x\\y\end{pmatrix} = 8x^2 - 4xy + 5y^2$ for each $(x,y) \in \mathbb{R}^2$. *(Marks 4)*

ii. Use the matrix A in part (a)i to find a change of variable that transform the ellipse $8x^2 - 4xy + 5y^2 = 180$ in to the form of $\frac{X^2}{a} + \frac{Y^2}{b} = 1$. Determine the values of a and b. *(Marks 8)*

(b) Use Lagrange Multipliers to find the closest point on the circle $x^2 + y^2 = 136$ from the point $(5, -3)$. *(Marks 8)*

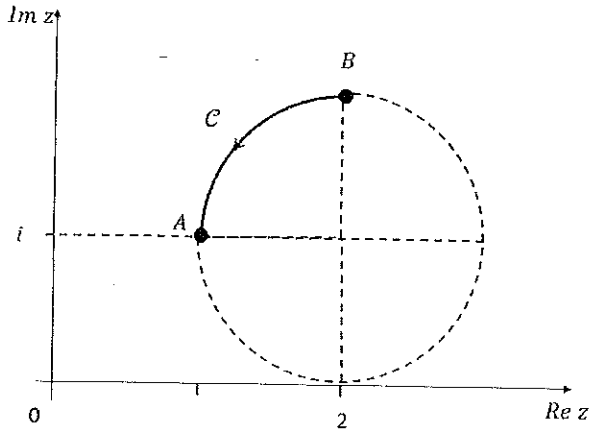
2. (a) Determine whether the limit of $\dfrac{\bar{z}-1}{z-1}$ as z approach to 1 exist or not. Find $\lim\limits_{z\to 1}\dfrac{\bar{z}-1}{z-1}$ if the limit exists. *(Marks 4)*

(b) Discuss the continuity of $f(z)=\begin{cases}\dfrac{z^2+1}{z-i} & \text{when } z\neq i\\ 1+2i & \text{when } z=i\end{cases}$ on $\mathbb{C}$. *(Marks 4)*

(c) Let $f(z)=z-\bar{z}$ for $z\in\mathbb{C}$. If $f(z)=u(x,y)+iv(x,y)$ for $z=x+iy\in\mathbb{C}$, prove whether the real valued functions u and v are harmonic on $\mathbb{R}^2$? *(Marks 6)*

(d) Find an analytic function g such that $Im(g)=2y$ and $g(1)=0$. *(Marks 6)*

3. (a) Consider the following image.



i. Parameterize the curve C which connects the point B, to the point A as in the above image. *(Marks 4)*

ii. Compute the integral of $f(z)=\dfrac{1}{z-2-i}$ along the curve C. *(Marks 8)*

(b) Find $\int_{\bar{C}}\dfrac{e^{2z}+\sin 3z}{z^2(z^2-3-4i)}dz$, where $\bar{C}$ is the positively oriented circle centered at $z=2+i$ with radius 1. *(Marks 8)*

4. The variables $x = f(t),\ y = g(t)$ satisfy the following system of first order differential equations.

$$\begin{cases} \dfrac{dx}{dt} &= x - 2y \\ \dfrac{dy}{dt} &= x - y. \end{cases} \tag{1}$$

(a) Show that $\dfrac{d^2x}{dt^2} + x = 0.$ *(Marks 4)*

(b) Determine whether $x_0 = 0$ is a regular singular point for the differential equation $\dfrac{d^2x}{dt^2} + x = 0.$ *(Marks 2)*

(c) Find the series solution of $\dfrac{d^2x}{dt^2} + x = 0$ around $x_0 = 0.$ *(Marks 10)*

(d) Hence find the solution of system (1). *(Marks 4)*

5. (a) Find a change of variable to eliminate the term u from the following equation. Justify your answer.

$$u_x + e^{2x}u_y + u = ye^{-x} \tag{2}$$

(Marks 8)

(b) Use the method of characteristics to find the solution of $u_x + e^{2x}u_y = y$ subject to the initial condition $u(0, y) = \dfrac{1}{4} + y.$ *(Marks 8)*

(c) Hence find the solution of equation (2). *(Marks 4)*

6. (a) Consider the initial value problem,

$$y' + (x + 1)y = 0 \quad \text{subject to } y(0) = 2. \tag{3}$$

Use the fourth order Runge Kutta method with step size $h = 0.1$ to evaluate the solution of equation (3) at $x = 0.1$ up to three decimal places. *(Marks 4)*

Hint: Formula of Fourth Order Runge Kutta method:

$$y_{n+1} = y_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right);\ \text{for } n = 0, 1, \cdots$$

iii. Find the critical value/s for this hypothesis testing at 1% significant level. (*Marks 1*)

iv. Test the hypothesis at 1% significant level and state your decision regarding the hypothesis testing. (*Marks 2*)

v. Interpret your findings in question (c)iv. (*Marks 2*)

vi. State the assumptions that you made for this hypothesis testing. (*Marks 1*)

9. Following summery quantities were given for a study of a investigating the relationship between roadway surface temperature (x in unit of $^\circ F$) and the pavement deflection(y).

$$n = 20,$$

$$\sum_{i=1}^{20} x_i = 1478, \ \sum_{i=1}^{20} y_i = 12.75,$$

$$\sum_{i=1}^{20} y_i^2 = 8.86, \ \sum_{i=1}^{20} x_i^2 = 143215.8, \ \sum_{i=1}^{20} x_i y_i = 1083.67.$$

(a) Calculate the correlation coefficient for above data. Interpret your answer of the value of correlation coefficient. (*Marks 7*)

(b) Find the equation of the best fitted simple linear regression model of the above data. (*Marks 7*)

(c) Interpret the found regression coefficient in part (b). (*Marks 2*)

(d) Use the equation of the fitted line to predict what pavement deflection would be observed when the surface temperature is $85^\circ F$. (*Marks 2*)

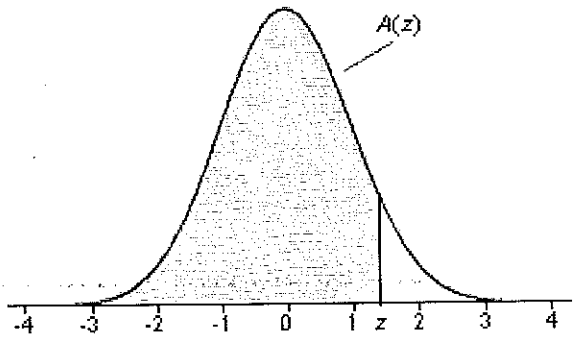
(e) Clearly state the assumptions you have made to find best fitted linear model. (*Marks 2*)

End.

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Z-table

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

t Distribution: Critical Values of *t*

Degrees of freedom	*Two-tailed test:* / *One-tailed test:*	*Significance level* 10% / 5%	5% / 2.5%	2% / 1%	1% / 0.5%	0.2% / 0.1%	0.1% / 0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291