The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical & Computer Engineering

Study Programme	: Diploma in Information Systems and Technology
Name of the Examination	: Final Examination
Course Code and Title	**: EEZ3562 Mathematics**
Academic Year	: 2020/21
Date	: 28th December 2021
Time	: 0930-1230hrs
Duration	: **3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Four (4)** pages.
3. Answer any **Five (5)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Relevant charts/ codes are provided.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.

[Q1]

(a) If a, b and c are Boolean variables, then simplify the following expressions using Demorgan's theorem. [06]

(i) $\overline{(\bar{a}+b+c)\overline{bc}}+a$

(ii) $\overline{(\bar{c}+\bar{a}+b)\overline{a+b}}$

(b) If x and y are Boolean variables, then tabulate the following expressions using Truth tables. [06]

(i) $\overline{(\bar{x}+\bar{y})(\bar{x}+y)}=x$

(ii) $(x+\bar{y})+(\bar{x}+y)=xy+\overline{x+y}$

(c) Minimize the following using the Karnaugh maps method. Hint: first construction the truth table and then find the simplified Boolean expression for the given expression. [08]

$$R=\overline{ABC}+\bar{A}B+AB\bar{C}+AC$$

[Q2]

(a) If A $=\begin{bmatrix}1 & -1\\ 2 & 0\end{bmatrix}$, show that $A^2+2I=A$, where I is the identity matrix of order 2. [06]

(b) Let $A=\begin{bmatrix}-1 & 1 & -1\\ 3 & -3 & 3\\ 5 & -5 & 5\end{bmatrix}$, show that $A^2=A$;

Hence deduce that $(I-A)^2=(I-A)$, where I the identity matrix of order 3. [08]

(c) Let $A=\begin{bmatrix}-5 & -8 & 0\\ 3 & 5 & 0\\ 1 & 2 & -1\end{bmatrix}$, show that $A^2=I$, where I is the identity matrix of order 3. [06]

[Q3]

Let $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$

(a) Find AA^T. [06]

(b) Find the inverse of the matrix A using Gaussian elimination method. [14]

[Q4]

(a) By using the integration by parts, find the indefinite integral. [06]

(i) $\int xcos3xdx$

(ii) $\int x^4 lnxdx$

(b) Evaluate following the definite integrals. [06]

(i) $\int_0^2 x(x^2-1)^7 dx$

(ii) $\int_1^4 \frac{x^3-8}{\sqrt{x}}\, dx$

(c) If $\int_0^{\frac{\pi}{2}} xsinx\, dx = \int_0^2 (kx^2+2x)\, dx$ find the value of $\boldsymbol{k}$. [08]

[Q5]

(a) A pulley belt of length 300 cm takes $2s$ to make a complete revolution. If the radius of the pulley is 150 mm, then find the angular velocity of a point on the rim of the pulley. [04]

(b) Find the value of [08]

(i) Sin x + Sin 2x + Sin 4x, when $x = 60^0$

(ii) Cos 4x - Cos 3x + Cos x, when $x = 120^0$

(c) Prove that [08]

$$\frac{Tan\theta}{1+Sec\theta} - \frac{Tan\theta}{1-Sec\theta} = 2Cosec\theta$$

[Q6]

(a) Evaluate the following limits. [06]

(i) $\lim_{x\to 0} \frac{sin5x}{3x}$ (ii) $\lim_{x\to 9} \frac{2x^2-162}{\sqrt{x}-3}$

(b) Let X and $\overline{X}$ are the exact value and the computed value of an answer respectively. Find the **absolute error** and the **relative error** when; [04]

(i) $X = -0.0047, \overline{X} = -0.0045$

(ii) $X = -0.671 \times 10^{12}, \overline{X} = -0.0669 \times 10^{13}$

(c) The difference table for $f(x) = e^x$ with $h = 0.2$ is shown below. [10]

x_i	f_i	Δf_i	$\Delta^2 f_i$	$\Delta^3 f_i$	$\Delta^4 f_i$
0.0	1.0000				
		0.2214			
0.2	1.2214		0.0490		
		0.2704		0.0109	
0.4	1.4918		0.0599		0.0023
		0.3303		0.0132	
0.6	1.8221		0.0731		0.0031
		0.4034		0.0163	
0.8	2.2255		0.0894		0.0033
		0.4928		0.0196	
1.0	2.7183		0.1090		0.0047
		0.6018		0.0243	
1.2	3.3201		0.1333		
		0.7351			
1.4	1.0552				

Select $x_0 = 0.4$ and using **Newton's forward** formula, find the value of $f(0.43)$.

[Q7]

(a)

(i) By using the differentiation of the first principles, find the gradient of $f(x) = 5x^2 - 3x + 7$ at the point $x = -2$. [04]

(ii) Evaluate the limit $\lim_{x \to 0} \frac{\sin\left(\frac{\pi}{4}+x\right) - \sin\left(\frac{\pi}{4}\right)}{x}$. [04]

(b) Find the first derivative of each of the following function. [06]

(i) $f(x) = \frac{x\,cosec(x)}{3 - cosec(x)}$

(ii) $f(x) = (2e^{x^2} + x^2)^3$

(c) Find the equation of the line perpendicular to the graph of $y = \frac{\tan x}{1-\tan x}$ at the point $x = \frac{\pi}{4}$. [06]

[Q8]

(a) A pyramid has a square base with side length of 8 units. The four lateral faces are congruent isosceles triangles with lateral edges of length 10 units. Find the surface area of this pyramid. [10]

(b) A trapezoid $ABCD$ with AD parallel to BC has angle D equal to 40 degrees, the length of DC is equal to 2 meters, the length of BC is equal to 5 meters and the area of the trapezoid is equal to 20 m^{-2}. Calculate the length of AD. [10]

----------End of the paper----------