The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: Five (05)
Name of the Examination	: Final Examination
Course Code and Title	**: PEU5300 – Riemann Integration**
Academic Year	: 2021/2022
Date	: 17.10.2022
Time	: 09.30 a.m. – 11.30 a.m.
Duration	: 2 hours
Index number	:

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Six (06)** questions in **Two (02)** pages.
3. Answer any **Four (04)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
5. Relevant log tables are provided where necessary.
6. Having any unauthorized documents/ mobile phones in your possession is a punishable offense
7. Use blue or black ink to answer the questions.
8. Circle the number of the questions you answered in the front cover of your answer script.
9. Clearly state your index number in your answer script

Q1)

Let $f(x) = \begin{cases} 1+x, & x \in [0,2] \cap \mathbb{Q} \\ 1, & x \in [0,2] \cap \mathbb{Q}^c \end{cases}$ and $g(x) = x^2, x \in [0,2]$.

By using the definitions of the Upper Riemann Integral and the Lower Riemann Integral of a bounded function defined on $[a,b]$, show that

$$\overline{\int}_0^2 f(x) = 4 \text{ and } \underline{\int}_0^2 g(x) = \frac{8}{3}.$$

Q2)

(a) Consider the function

$$f(x) = \begin{cases} 1, & -1 \le x < 0 \\ 0, & x = 0 \\ 1, & 0 < x \le 1 \end{cases}.$$

By using the Riemann's Criterion for integrability, show that the function f is integrable on $[-1,1]$.

(b) Let g be a continuous function on $[a,b]$. Prove that g is Riemann integrable on $[a,b]$

Q3)

(a) Let f and g be Riemann integrable functions on $[a,b]$ such that for each $x \in [a,b]$, $f(x) \le g(x)$. Prove that $\int_a^b f(x)dx \le \int_a^b g(x)dx$

(b) Let h be a Riemann integrable function on $[a,b]$. Deduce from (a) that, if for each $x \in [a,b]$, $m \le h(x) \le M$ for some $m, M \in \mathbb{R}$, then $m(b-a) \le \int_a^b h(x)dx \le M(b-a)$, and that $\left|\int_a^b h(x)dx\right| \le \int_a^b |h(x)|dx$.

(c) Assuming the function h defined by $h(x) = \left(x + \frac{1}{x}\right)^x, x \in [1,2]$ is Riemann integrable on $[1,2]$, show that $2 \le \int_1^2 \left(x + \frac{1}{x}\right)^x dx \le \frac{25}{4}$.

Q4)

State the First Version of Fundamental Theorem of Calculus.

By using the above Theorem, find

(a) $\lim_{x\to 0}\frac{1}{x}\int_0^x e^{t^2}\,dt$,

(b) $F'(x)$, where $F(x)=\int_{x^2}^{x^3} e^t\cos t\,dt$, and

(c) all continuous functions f such that $f(x)\neq 0, \forall x$ and $\int_0^x f(t)dt = (f(x))^2 + C$, where $C \in \mathbb{R}$,

Q5)

(a) Evaluate the indefinite integral of $F(t)=\int_0^1 \frac{x^t-1}{\ln x}\,dx$, for $t \geq 0$, by first evaluating $F'(t)$.

(b) Using the properties of Riemann Sums, find an integral of a function over an appropriate interval that is represented by

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{5}{n}\right)\left[5\left(\frac{5i}{n}-2\right)^3-1\right].$$

Hence find the above limit.

Q6)

Consider the following integrals:

(i) $\int_1^2 \frac{x^2+x+1}{(x^2-1)^{1/3}}\,dx$,

(ii) $\int_0^\infty \frac{1}{\sqrt{x^{1/3}+x^3}}\,dx$,

(iii) $\int_1^\infty \frac{\cos x}{x^2+1}\,dx$,

(iv) $\int_{-\infty}^{-1} \frac{e^x}{x}\,dx$.

(a) Explain why each of the above is an improper integral.

(b) Determine the convergence of each of them. Justify your answers.

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