The Open University of Sri Lanka
B.Sc. Degree Programme, Level – 04
Final Examination – 2021/2022
PHU4303 – Mathematical Methods for Physics
Duration: 2 hours

Date: 27th October 2022 **Time: 9.30 a.m. to 11.30 a.m.**

This paper has 3 pages and 6 questions. **Answer any four (4) questions**
Non-programmable calculators are allowed.

1.Read the following description and answer the questions given below.
A farmer met a cunning magician at the village market. Magician said he has a magic hat that can double the amount of money put in to it. But to do that once, he charge a fee of 500 rupees. The farmer agreed to this magic trick and put x amount of money in to the hat. Then magician ~~asks~~ asked the farmer to close the eyes and count to 100. At the end of the counting farmer opened the eyes. He found the amount in the hat had 'magically' doubled. He paid the fee of 500 rupee to the magician. He put the remaining money back in to the hat and asked to double it. It doubled again and farmer paid the fee. He did this for the third time. But after paying the fee for the third time farmer lost all the money.

a. What was the starting amount (x) farmer put in to the hat? (5 marks)

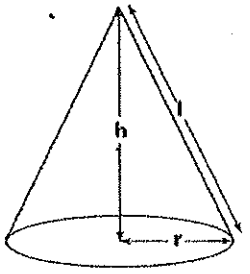
b. Another farmer agreed with the magician to do the same. Magician doubled his money and he paid 500 for each attempt. After doing the trick for 5 times, the farmer still had the same amount of money as started. What was his starting amount? (5 marks)

c.Another magician agrees to double the amount for a fee of 2/3rd of the final amount.

i. If the amount put in to the hat is x_n and the amount left at hand after doubling and paying the fee is x_{n+1} Find the relationship between x_n and x_{n+1} (5 marks)
ii. Using your knowledge in series and sequences show that no matter what the starting amount farmer would run out of money if he keeps on playing. (10 marks)

2.

a. A person made a cone of base radius is 'r' and height is 'h'. Its slanted length is 'l'.



i.Find the relationship between r,h and l. (5 marks)

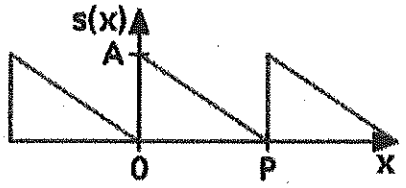
ii.The volume of the cone is given by $\frac{1}{3}\pi r^2 h$. If the slanted length 'l' is fixed to 10 units, Find the r and h, which provides the largest volume. (10 marks)

b.

i.Show that the minimum/maximum of the graph $ax^2 + bx + c = 0$ is given by $x = -\frac{b}{2a}$ (5 marks)

ii.Calculate the area under the curve $f(x) = x^3 + 10x + 5$ from $x = 4$ *to* 8 (5 marks)

3.Consider the following signal



a. What is the period of the signal? (3 marks)

b.Write an expression describing the signal for a single period. (5 marks)

c.Write the Fourier Series equation. (5 marks)

d.If the period is equal to π,calculate the Fourier Co-efficiencies of the above signal. (12 marks)

4.

a.How do you test if a differential equation is exact? (10 marks)

b.Newton's law of cooling: "Temperature of a body changes at a rate proportional to the difference between the temperature of the body and the temperature of the surrounding medium." If T_m is the temperature of the medium and $T = T(t)$ is the temperature of the body at time t, then

$$\frac{dT}{dt} = -k(T - T_m)$$

Where k is a positive constant and the minus sign indicates; that the temperature of the body increases with time if it is less than the temperature of the medium, or decreases if it is greater.

i. Solve the above differential equation (5 marks)

ii.The temperature of the surrounding medium is 25°C. Assume it is not changing. The initial temperature of the body is 75C. It takes 90 seconds to cool it to 60°C. How long will it take to cool to 45°C? (10 marks)

5.

a.A vector field is described by the following equation.

$$3x\,\boldsymbol{i} - 5x^2\,\boldsymbol{j} + 10zx\,\boldsymbol{k}$$

i.Calculate the curl of the above vector field. (10 marks)

ii.Calculate the divergence of the above vector field. (5 marks)

b.Calculate the gradient of the following scalar field. (10 marks)

$$3x - 5x^2z + 10zx$$

6.

a.

i.Find the determinant of the following matrix. (10 marks)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

ii.Find the eigen value of the following matrix. (10 marks)

$$A = \begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix}$$

b.Let $f(x, y, z) = \sin(2x) + 3xy - z^2$. Calculate partial derivate with respect to x and y. (5 marks)

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