The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: 04
Name of the Examination	: Final Examination
Course Code and Title	: PEU4302- Linear Algebra
Academic Year	: 2021/2022
Date	: 08.10.2022
Time	: 9.30 a.m.-11.30 a.m.
Duration	: 2 Hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 06 questions in 03 pages.
3. Answer any 04 questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Having any unauthorized documents/ mobile phones in your possession is a punishable offense.
6. Use blue or black ink to answer the questions.
7. Circle the number of the questions you answered in the front cover of your answer script.
8. Clearly state your index number in your answer script.

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1. (a) Show that a square matrix is invertible if and only if it is non-singular.

(b) Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$.

(c) Find a matrix A satisfying the matrix equation $A\begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$.

(d) Find the inverse of the following matrix:

$$\begin{pmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{pmatrix}.$$

2. (a) Find a row reduced echelon form of the matrix A, where

$$A = \begin{pmatrix} 0 & 2 & 4 & 0 \\ 1 & -1 & 0 & 1 \\ 2 & 1 & 2 & 1 \\ 0 & 3 & 2 & -1 \end{pmatrix}$$

Hence find the rank of A.

(b) Find non- singular matrices P and Q such that PAQ is normal, where

$$A = \begin{pmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{pmatrix}.$$

(c) Verify the Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}.$$

3. (a) Define each of the following terms:

(i) A consistent system and an inconsistent system.

(ii) Homogeneous system and non- homogeneous system.

(b) Solve the following system of homogeneous equations:

$$x+y-2z+3w=0$$
$$x-2y+z-w=0$$
$$4x+y-5z+8w=0$$
$$5x-7y+2z-w=0.$$

(c) Find for what values of λ and μ the system of linear equations

$$x+y+z=6$$
$$x+2y+5z=10$$
$$2x+3y+\lambda z=\mu$$

has

(i) a unique solution,

(ii) no solution,

(iii) infinite number of solutions. Also find the solution in this case.

4. (a) Check whether the following matrix is diagonalizable or not. Justify your answer and diagonalize if possible.

$$\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$

(b) Transform the following quadratic form to canonical form by an orthogonal transformation:

$$-5x_1^2+x_2^2-x_3^2+6x_1x_3+4x_1x_2$$

5. Solve each of the following systems of linear equations by LU-decomposition:

(a) $x+2y+3z=6$

$2x+3y+z=9$

$3x+y+2z=8$

(b) $-x_1 + x_3 = 5$

$2x_1 + 3x_2 - 2x_3 + 6x_4 = -1$

$-x_2 + 2x_3 = 3$

$x_3 + 5x_4 = 7$

6. (a) (i) Prove that any square matrix A can be written as the sum of a Hermitian matrix and a skew-Hermitian matrix.

(ii) Show that the following matrix is Hermitian:

$$A = \begin{pmatrix} 3 & 2+i \\ 2-i & 7 \end{pmatrix}.$$

(iii) Find its eigen values and eigen vectors.

(b) (i) Define the Hermitian Form of a matrix.

(ii) Find the Hermitian form of

$$A = \begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & i \\ 2 & -i & 0 \end{bmatrix} \text{ with } X = \begin{bmatrix} i \\ 1 \\ -i \end{bmatrix}.$$