The Open University of Sri Lanka
Department of Mathematics
B. Sc/ B. Ed Degree Programme
Open Book Test - 2021/ 2022
Pure Mathematics– Level 03
PEU3301 – Foundations of Mathematics
Duration: One Hour

Date: 27.08.2022

Answer All Questions

Q1)

(i) Let $B_n = \{x \in \mathbb{Z}: -n \leq x \leq n\}$.
Determine $\bigcup_{i=1}^{3} B_n$, $\bigcup_{i=3}^{\infty} B_n$, $\bigcap_{i=1}^{3} B_n$, and $\bigcap_{i=3}^{\infty} B_n$.

(ii) Let $A = \{2l: l \in \mathbb{Z}\}, B = \{3m: m \in \mathbb{Z}\}$, and $C = \{6n: n \in \mathbb{Z}\}$.
Prove the identity $A \cap B = C$.

(iii) Let $P = \{1,2,3,4,5,6,7,8,9,10\}$,
$Q = \{m^2: m \in \mathbb{N}\}$, and $R = \{2n: n \in \mathbb{N}\}$. List the elements of
$P \cap (R \setminus Q) \cup (Q \setminus R))$, where $R \setminus Q = \{x : x \in R \text{ and } x \notin Q\}$.

Q2) Let $\mathcal{R}$ be the relation defined on $\mathbb{Z}$ by $x\mathcal{R}y$ if 2 divides $x + y$ for $x, y \in \mathbb{Z}$.

(i) List five numbers that are related to $x = 2$.

(ii) Show that $\mathcal{R}$ is an equivalence relation on $\mathbb{Z}$.

(iii) Find the equivalence class of -3.

Q3) Let $g(x) = \frac{x+4}{2x-5}, x \in \mathbb{R} \setminus \left\{5/2\right\}$. Show that g is a bijection from $\mathbb{R} \setminus \left\{5/2\right\}$ into $\mathbb{R} \setminus \left\{1/2\right\}$. Find g^{-1}.

Q4) Let g and h be the functions defined on $\mathbb{R}$ by

$g(x) = 9 + 6x + x^2$ and $h(x) = x^2 + 1$. Find $g(A)$ and $h^{-1}(B)$,
where $A = \{x \in \mathbb{R}: x \leq 1\}$, and $B = \{y \in \mathbb{R}: 5 \leq y \leq 10\}$. Justify your answers.

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