The Open University of Sri Lanka

B.Sc/B.Ed. Degree Programme – Level 04

Open Book Test – 2021/2022

Pure Mathematics

PEU4303 – Group Theory 1

Duration: - One Hour.

Date: - 14.01.2023 **Time: - From 01.00 p.m. to 02.00 p.m.**

Answer All Questions

(01) (a) Show that $s_y \circ r_{90}^i$ for $i = 0,1,2$ and 3 is the complete list of reflections.

(b) Show that $r_{90} \circ s_y \circ r_{90} = s_y$, and hence prove that

$r_{90} \circ s_y = s_y \circ r_{90}^3$

(c) Prove that $r_{90}^k \circ s_y = s_y \circ r_{90}^{3k}$, where $k \in \mathbb{N}$.

(d) Express the following symmetries in the form $s_y^j \circ r_{90}^i$.

(i) $(s_y \circ r_{90}) \circ (s_y \circ r_{90}^3)$ (ii) $r_{90}^2 \circ (s_y \circ r_{90}^2)$

(iii) $s_x \circ r_{90}^{-3}$ (iv) $r_{90}^{-5} \circ s_x$

(02) (a) Which of the following sets of matrices are closed with respect to matrix multiplication? Justify your answer.

(i) $\left\{\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} : x \in \mathbb{R}\right\}$

(ii) $\left\{\begin{pmatrix} x & 1 \\ 1 & 0 \end{pmatrix} : x \in \mathbb{R}\right\}$

(b) Consider the binary operation $\circ$ defined on $\mathbb{Z}$ by $a \circ b = a + 2b$.

Is operation $\circ$ associative on $\mathbb{Z}$?

(c) Let $\circ$ be the operation defined on $\mathbb{Z}$ by $a \circ b = ab^2$.

Find the identity element if there exists. Justify your answer.

(d) Find inverses of 1,2 and 3 in $(\mathbb{Z}_4, +_4)$.