The Open University of Sri Lanka

B.Sc/B.Ed. Degree Programme – Level 04

CAT 2 Online– 2021/2022

Pure Mathematics

PEU4303 – Group Theory 1

Duration: - One Hour.

Date: - 05.02.2023 **Time: - From 09.00 a.m. to 10.00 a.m.**

Answer All Questions

(01) (a) Show that $\mathbb{Z}_7^{\times} = \{1,2,3,4,5,6\}$ with respect to multiplication mod 7 forms a group.

(b) Find the order of each of the elements 4, 5, 6 in the group $(\mathbb{Z}_7^{\times}, \times_7)$.

(c) Prove that the group $(\mathbb{Z}_{11}^{\times}, \times_{11})$ is cyclic. Find all the generators.

(02) (a) Find all proper subgroups of $(\mathbb{Z}_{12}, +_{12})$.

(b) Express each of the following permutations as single cycles or as the product of disjoint Cycles. Hence find the order of each.

(i) (1 2)(2 3 6)
(ii) (2 3 4)(1 3 5)(2 5)
(iii) (1 3 2 6)(1 2 4)(3 5)
(iv) (3 5 6)(1 6)(2 3 4).