The Open University of Sri Lanka
Department of Mathematics
B. Sc/ B. Ed Degree Programme
CAT 2 - 2021/ 2022 (Online Test)
Pure Mathematics– Level 05
PEU5301 – Ring Theory
Duration: One Hour

Date: 10.02.2023 **Time: 2.30 p.m. - 3.30 p.m.**

ANSWER All Questions

Q1) Let R be a commutative ring with unity and $a \in R$. Prove that

(i) $I = \{ar + na : r \in R, n \in \mathbb{Z}\}$ is an ideal of R and
(ii) $I = < a >$.

Q2)

(i) Show that $p(x) = x^3 + x^2 + 2$ is an irreducible polynomial in $\mathbb{Z}_3[x]$.
(ii) By clearly stating any results you use, deduce that $\mathbb{Z}_3[x]/I$ is a field, where $I = < p(x) >$
(iii) How many elements are in the field $\mathbb{Z}_3[x]/I$? Justify your answer.

Q3) Let $m, n \in \mathbb{Z}$ and $I = < m - ni >$ be the ideal in $\mathbb{Z}[i]$. Let $\phi: \mathbb{Z}[i] \to \mathbb{Z}[i]/I$ be defined by

$$\phi(a + ib) = (a - bi) + I \text{ for } a, b \in \mathbb{Z}.$$

Show that

(i) ϕ is a ring homomorphism,
(ii) ϕ is surjective,
(iii) $\ker \phi =< m + ni >$, and
(iv) $\mathbb{Z}[i]/< m + ni > \cong \mathbb{Z}[i]/< m - ni >$ (Clearly state any results, you use)

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