00040

The Open University of Sri Lanka

B.Sc./ B.Ed. Degree Programme

Pure Mathematics – Level 05

PEU5303 – Number Theory

Open Book Test (OBT) – 2021/2022

Duration: One (01)–hour

Date: 21.01.2023 **Time: 09.00 a.m. -10.00 a.m.**

ANSWER ALL QUESTIONS.

(01) (a) Prove that $\sqrt{\frac{2}{3}}$ is not rational.

(b) Prove that for each $n \in \mathbb{N}$, $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

(c) Let a, b and c be three positive integers. Prove that

(i) if $(a,b) = d$ then $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$,

(ii) $(a, bc) = 1$ if and only if $(a,b) = (a,c) = 1$.

(02) (a) Find $x, y \in \mathbb{Z}$ s.t $(3587, 1819) = 3587x + 1819y$,

(i) Hence find $[3587, 1819]$.

(b) Consider the Diophantine equation $21x + 14y = 70$.

(i) Show that an integer solution exists for the equation,

(ii) Find a particular solution, $x = x_0, y = y_0$,

(iii) Write down the complete solution in terms of x_0 and y_0.

(c) By using the theory of congruence verify that $27 | 2^{5n+1} + 5^{n+2}$.