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The Open University of Sri Lanka
Department of Mathematics
B.Sc./B.Ed. Degree Programme
Open Book Test (OBT) - 2023/2024
Pure Mathematics - Level 05
PEU5304 - Introduction to Complex Analysis

Duration: One Hour

Date: 16.07.2023 **Time: 1.00 p.m. - 2.00 p.m.**

ANSWER ALL QUESTIONS.

1. Express the following complex numbers in the standard form $a + ib$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.

   (a) $\frac{5i}{(1-i)(2-i)(3-i)} + \frac{2-i}{5i}$,

   (b) $\frac{i^{4n+3}}{i^{4n-1} + i^{4n}}$, where $n \in \mathbb{Z}^+$,

   (c) $\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{17}$.

2. (a) Let $z_1, z_2, z_3, z_4 \in \mathbb{C}$. Prove that $\frac{Re(z_1+z_2)}{|z_3+z_4|} \leq \frac{|z_1|+|z_2|}{||z_3|-|z_4||}$ when $|z_3| \neq |z_4|$.

   (b) Show that the hyperbola $x^2 - y^2 = 1$ can be written as $z^2 + \bar{z}^2 = 2$, where $z = x + iy$ with $x, y \in \mathbb{R}$, $i^2 = -1$ and $\bar{z}$ is the conjugate of z.

3. If $z_1, z_2, z_3 \in \mathbb{C}$ are the vertices of an equilateral triangle, show that

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1.$$