00159

THE OPEN UNIVERSITY OF SRI LANKA

B.Sc./B.Ed DEGREE PROGRAMME – LEVEL 04

FINAL EXAMINATION -2023/2024

PURE MATHEMATICS

PEU4302/PEE4302 – Linear Algebra

DURATION: Two (02) Hours

Date: 03.10.2023 **Time:9.30 a.m.-11.30 a.m.**

Answer FOUR questions only.

1. (a) Define each of the following:

(i) Inverse of a matrix.

(ii) Equivalent matrices.

(iii) Rank of a matrix

(iv) Non-singular matrix.

(v) If A and B are square matrices such that $AB = I$ show that A is non-singular and $A^{-1} = B$.

(b) Without expanding the determinant, prove that $\begin{vmatrix} \frac{1}{a} & a & bc \\ \frac{1}{b} & b & ca \\ \frac{1}{c} & c & ab \end{vmatrix} = 0$.

(c) Find matrix A by solving the following matrix equation:

$$A\begin{pmatrix} 1 & 2 \\ 5 & -1 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -2 & -4 \end{pmatrix}.$$

(d) Find the inverse of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & -2 & 0 & 0 \\ 3 & 1 & -2 & 0 \\ 1 & -1 & 3 & 3 \end{pmatrix}.$$

2. (a) Find the row reduced echelon form of the following matrix:

$$\begin{pmatrix} 1 & -2 & 1 & -1 \\ 1 & 1 & -2 & 3 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{pmatrix}.$$

(b) Find the rank of the matrix A, where $A = \begin{pmatrix} 1 & 2 & -5 & 2 \\ 3 & -1 & 2 & 4 \\ 2 & 3 & -4 & 3 \\ 5 & 2 & -2 & 7 \end{pmatrix}$.

(c) Verify the Cayley – Hamilton Theorem for the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. Hence find A^{-1} and A^4. .

3. (a) Find non- singular matrices P and Q such that PAQ is normal, where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{pmatrix}.$$

(b) Solve the system of equations,

$$\begin{aligned} 6x_3 + 2x_4 - 4x_5 - 8x_6 &= 8 \\ 3x_3 + x_4 - 2x_5 - 4x_6 &= 4 \\ 2x_1 - 3x_2 + x_3 + 4x_4 - 7x_5 + x_6 &= 2 \\ 6x_1 - 9x_2 + 11x_4 - 19x_5 + 3x_6 &= 1. \end{aligned}$$

(c) Find values of λ for which the following system of equations is consistent and has non-trivial

solutions. Solve equations for all such values of λ.

$$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$$
$$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$$
$$2x+(3\lambda+1)y+3(\lambda-1)z=0.$$

4. (a) If $X_1=\frac{1}{3}\begin{bmatrix}2 & -1 & 2\end{bmatrix}^T$ and $X_2=k\begin{bmatrix}3 & -4 & -5\end{bmatrix}^T$ where $k=\frac{1}{\sqrt{50}}$, construct an orthogonal matrix $A=\begin{bmatrix}X_1 & X_2 & X_3\end{bmatrix}$.

(b) Transform the quadratic form $-5x_1^2+x_2^2-x_3^2+6x_1x_3+4x_1x_2$ to canonical form by orthogonal transformation. Write the orthogonal transformation which reduces the Quadratic form to Canonical form.

5. (a) Show that the matrix $\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}$ is diagonalizable.

Hence, find the corresponding diagonal matrix.

(b) Solve the following system of equations by LU-decomposition:

$$x+y+z+w=5$$
$$2x+y-z-2w=3$$
$$x-y+2z+w=3$$
$$x-3y+z+w=9.$$

6. (a) (i) Define the Hermitian form of a matrix.

(ii) Find the Hermitian form of

$$A=\begin{pmatrix}4 & 2-i & 5+2i\\ 2+i & 1 & 2-5i\\ 5-2i & 2+5i & 2\end{pmatrix} \text{ with } X=\begin{pmatrix}1\\ i\\ -i\end{pmatrix}.$$

(b) Verify that the matrix $A=\frac{1}{2}\begin{pmatrix}1+i & 1-i\\ 1-i & 1+i\end{pmatrix}$ has eigen values with unit modulus.

(c) Find the eigen values and eigen vectors of the following matrix A.

$$A=\begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$