THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Industrial Studies

683

Final Examination (2021/2022)
MHZ3458 Mathematics for Agriculture

Date: 8th February 2023 (Wednesday) **Time: 0930 hrs. –1230 hrs.**

Instructions:

- *Answer **five (05)** questions only.*
- *Number of pages in the paper is six (06).*
- *All the symbols are in standard notation unless they are defined.*

Q1.

a. Simplify the following expressions

i. $x - 3y + 9z + 5x - 9z + 3y - 2x - 7y + 6z$ (10%)

ii. $8(5z - y) + 9y - 3(5y - 6z)$ (10%)

iii. $3p^3 + 5pq^2 - 7q^2 + q - 2p^2q + 5q^2 - 8p^3 + 5p^2q + 2q^2 + 5p^3$ (10%)

iv. $$\frac{12^{-\frac{1}{3}} . a^{\frac{5}{2}} . b^{\frac{1}{4}}}{96^{-\frac{1}{3}} . \left(a^{\frac{1}{2}} . b^{-\frac{3}{4}}\right)^5}$$ (10%)

b. Expand and simplify the following expressions.

i. $(-3x^2 + 2x)(x^2 - 5) - (2x + 1)(3x^2 + 6).$ (15%)

ii. $(x + 2)(5 - x) + (2x + 1)(x - 1)$ (15%)

c. If the simplified expression of ***b (i) (Dividend)*** is divided by the simplified expression of ***b(ii) (Divisor), then*** find the ***Quotient*** and the ***Remainder*** using the polynomial long division method. (30%)

Q2.

a. Factorize the following expression.

$$20ax^3 - 5a^3x + 10a^2x^2 - 5a^3x$$

i. What is the monomial factor of the expression? (5%)

ii. What are the other factors of the expression? (5%)

b.

i. Find the LCM (Least Common Multiple) of the following three expressions. (25%)

$(36p^2 - q^2)$, $(24p - 4q)$, $(20q^2)$

ii. Hence, simplify the below expression.

$$\frac{pq}{36p^2 - q^2} - \frac{q-p}{24p-4q} + \frac{q^2}{20q^2}$$ (25%)

c. Solve the following system of linear equations and find the values of x, y, z. (40%)

$$\begin{aligned} 2x + 6y + z &= 0 \\ -9x + 5y - 3z &= 8 \\ 5x - 7y + 9z &= -12 \end{aligned}$$

Q3.

a.

i. Solve the following quadratic equation by using the completing square method.

$3x^2 - 8x = 5$ (20%)

ii. Use the factorizing method to solve the following quadratic equation.

$-3x^2 + x + 2 = 0$ (20%)

b. Solve the following inequality.

$$\frac{2}{(x-1)} \geq 3$$ (20%)

i. Represent the solution on a number line. (10%)

ii. Write the answer in set notations. (10%)

a. Find the values of the following without using the logarithm tables.

i. $\log_2 3 \ \log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8$ (10%)

ii. $2\left[\frac{\log\sqrt{32}+\log 1000-\log\sqrt{243}}{\log 8+3\log 5}\right] + \frac{5}{3}\log 15$ (10%)

Q4.

a. Write the following sets in the builder form.

i. $\{2,4,6,8,10,12,14,16,18\}$ (10%)

ii. $\{1,3,5,7,11,13\}$ (10%)

b. Let $U = \{1,2,3,4,8,9,10\}$, $A = \{2,4,8\}$, $B = \{1,8,9,10\}$, and $C = \{3,8,9\}$. Find the following sets.

i. $(A \cap B) \cup (B \cap C)$ (10%)

ii. $C - B$ (10%)

iii. $B - A$ (10%)

c. The number of students registered for BIS Agriculture Degree at OUSL in 2023 is 65. Only the courses MHZ3458, AGI3553 and AGI3551 are offered for BIS Agriculture Degree for year 2023 and each student is registered at least for one of the courses. The number of students registered for different courses are as follows.

Course Code	Number of students registered
MHZ3458	40
AGI3551	42
AGI3553	48
Both MHZ3458 and AGI3551	30
Both AGI3551 and AGI3553	25
MHZ3458, AGI3551, AGI3553	20

i. Draw a Venn Diagram representing the above. (25%)

ii. Calculate the number of students registered for AGI3553 but not MHZ3458. (25%)

Q5.

a. Find the number of terms and the summations of the following series.

i. $\frac{1}{6}+\frac{1}{3}+\frac{2}{3}+\frac{4}{3}+\ldots\ldots\ldots\ldots+\frac{1024}{3}$ (20%)

ii. $-4+1+6+11+16+\cdots\ldots\ldots\ldots+236$ (20%)

b. Find the partial fractions of,

$$\frac{3x+5}{(x^2+5x+4)}$$ (35%)

c.

i. Write the general formula for binomial coefficients of the expansion of $(a+b)^n$. (5%)

ii. Using the formula stated in c(i) find the coefficient of the term containing x^3 of $(3x-2)^5$. (10%)

iii. Expand $(3x-2)^5$ using the binomial theorem. (10%)

Q6.

a. Prove that $\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2 = 1 - \sin x$ for $-\pi < x \leq \pi$. Hence, prove that the value of $\cos\frac{\pi}{12} - \sin\frac{\pi}{12} = \frac{1}{\sqrt{2}}$. (20%)

b. Prove that,

$$sin\,2\theta = \frac{2\tan\theta}{1+\tan^2\theta} \quad \text{for } 0<\theta<\pi \qquad (20\%)$$

c.

i. State the sine rule. (10%)

ii. Find the side lengths AC, BC and the angle $B\hat{A}C$ of the triangle given in ***Figure 6.1.*** (50%)

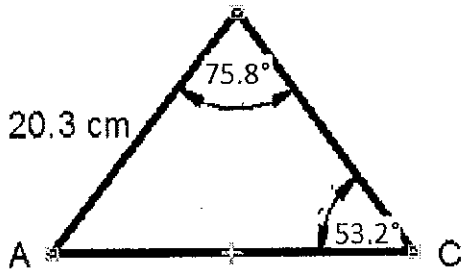


Figure 6.1

Q7.

a. Evaluate the following limits.

i. $\lim_{x\to 5} \frac{x^5 - 5^5}{x-5}$ (20%)

ii. $\lim_{x\to 3} \frac{x^2+2x-15}{x^2-2x-3}$ (20%)

iii. $\lim_{x\to 0} \frac{x^3}{sinx}$ (20%)

b. Differentiate the following expressions with respect to x.

i. $y = e^{2x} + 2xcosx + 2$ (20%)

ii. $y = \log\sqrt{\frac{2x}{x^2+1}}$ (20%)

Q8.

a. Find the following indefinite integrals.

i. $\int (x^5 + x^{-2} - 3)\, dx$ (20%)

ii. $\int \left(\frac{2sin\theta}{cos\theta - 2}\right) d\theta$ (20%)

b. Using the integration by parts, find the indefinite integral $\int (xsinx)dx$. (30%)

c. Integrate $\int (x + \frac{1}{x})\, dx$. Hence, find the area of the region bounded by the curve $y = (x + \frac{1}{x})$ the lines $x = 2$, $x = 4$ and the x axis. (30%)

Q9.

a. Let $\lambda, \mu \in \mathbb{R}$, $A = \begin{bmatrix} -4 & 2 \\ 7 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 8 \\ -1 & 3 \end{bmatrix}$ and $X = \begin{bmatrix} \lambda & \mu \\ \mu + 3 & 2\mu + \lambda + 1 \end{bmatrix}$. Find the values of λ and μ such that $AX + B = O_2$, where O_2 is the zero matrix of order 2. (40%)

b. If $C = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$, then find the inverse matrix of C. (20%)

c. Using the properties of determinants, prove that,

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 + a & ab + b & ac + c \end{vmatrix} = 0$$ (40%)

Q10.

a. Find the resultant force in the system of coplanar forces given in the ***Figure 10.1.*** (50%)

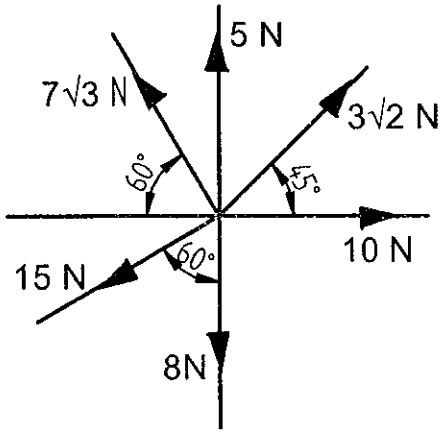


Figure 10.1

b. A uniform rod AB of mass 15kg and length 8 m is kept in horizontal level by the vertical supports at A and C. The length of the BC is 1 m. A mass of 10 kg is hung at B. Find the direction and the magnitude of the thrust that should be applied at A and C to preserve the equilibrium of the rod. (50%)

Q11.

a. Two particles of mass 3kg and 6 kg are connected by a light inextensible string which passes over a smooth fixed pulley. If the system is released from rest with the string taut and both particles are at a height of 1 m above the ground.

i. Find the acceleration of the system. (25%)

ii. Find the tension of the string. (25%)

iii. Find the greatest height reached by the 3 kg mass assuming that the 3 kg mass does not reach the pulley and that 6 kg mass does not rebound when it hits the ground. (30%)

iv. Find the work done by the 3 kg mass when reaching the maximum height. (20%)

End

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