THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering
Bachelor of Technology Honors in Engineering

Final Examination (2021/2022)
MPZ3552: Engineering Mathematics II

Date: 12th February 2023 (Sunday) **Time: 13:30 – 16:30**

Instruction:

- **Please answer a total of five (05) questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

Important integrals

- $\int e^{ax} sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$
- $\int e^{ax} cosbxdx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$
- $\int f(x)e^{ax}\,dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax}$ …… [Sign alternate $(+ - + - + \cdots)$]
- $\int f(x)cosax\,dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax - \cdots$ [Sign alternate in pairs $+ + - - + + - - \cdots$)]
- $\int f(x)sinax\,dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax - \cdots$ [Sign alternate in pairs after the first term $(+ + - - + + - - + + - - \cdots)$]
- $\int secax\,dx = \frac{1}{a}ln|secax + tanax|$

SECTION – A

Q1.

I. An organization decides to administer each of its 29 employees a skills test to see if any of them need to be retrained. The maximum attainable score on this test is 50, and the results are presented using an ordered stem-and-leaf diagram.

0	**5, 6**
1	**9, 9, 9**
2	**1, 6, 8, 8**
3	**3, 4, 5, 7, 9**
4	**1, 2, 3, 4, 4, 5, 6, 9, 9**
5	**0, 0, 0, 0, 0, 0**

a) Justify the applicability of a stem-and-leaf diagram to present this data. **[10%]**

b) Determine interquartile range of the scores. **[15%]**

c) The organization decides to retrain workers whose scores fall below the outside 1.5 times the inter quartile range below the lower quartile.

i. Verify that only two workers will undergo retraining. **[10%]**

ii. Create a suitable labeled box plot for this data, clearly indicating any outliers, as formed in part c). **[10%]**

iii. Discuss the skewness of the scores. **[05%]**

II. Over the time of four years, the bank manager maintains weekly statistics of the number of invalid cheques presented for the payment. The results are as follows:

Number of invalid cheques(x)	1	2	3	4	5
Number of weeks(f)	8	9	20	15	12

a) Find the central tendency measures (mean, median and mode) of the number of invalid cheques per week. [25%]

b) Which would you prefer as a suitable measure of central tendency. Justify your answer [15%]

c) Find the standard deviation of the number of invalid cheques per week. [10%]

Q2.

I. In a video game, the goal is to gather four jewels. The discrete random variable X represents the number of jewels collected by Saman in a single gameplay. The probability mass function of X is defined by,

$$P(X = x) = f(x) = \begin{cases} a, & x = 1; \text{a is a constant} \\ \frac{1}{2x}, & x = 2,3,4. \end{cases}$$

a) Find the value of a. [15%]

b) Determine the mean and the variance of the random variable X. [20%]

c) The random variable which represents the number of points a Saman receives for collecting X jewels is given by Y, where, $Y = 25X - 3$. Derive the mean and the standard deviation of the random variable Y. [10%]

II. Each morning, Mala intends to catch the staff transport to work. The vehicle is scheduled to depart at 7: 30 a.m. The number of minutes after 7: 30 a.m. in which the vehicle departs can be modeled by the random variable X, which has a continuous uniform distribution over the interval $[p, q]$, where $0 \leq p < q$.

a) Obtain the probability that Mala will catch the vehicle if she is 5 minutes. late at 7.30a.m. [10%]

b) If the mean and the variance of the random variable X is 4 and 3 find the values of p and q. [15%]

c) The probability that Mala oversleeps each morning is 0.04. If Mala oversleeps, she will definitely be late for work. If she does not oversleep, she will catch the vehicle on time, but she will be late for work if the vehicle departs at 7.35 a.m.

i. Find the probability that Mala will be late for work. [15%]

ii. Given that Mala is late for work, determine the probability that she overslept. [15%]

SECTION – B

Q3.

I. Define the vector product of the two vectors and determine the value of l so that, $2\,\underline{i} + l\,\underline{j} - \underline{k}$ and $-4\,\underline{i} - 2\,\underline{j} + 2\,\underline{k}$ are perpendicular. [15%]

II. Determine a unit vector which is perpendicular to plane $2\,\underline{i} - 6\,\underline{j} - \underline{3k}$ and $4\,\underline{i} + 3\,\underline{j} - \underline{k}$. [15%]

III. Let $\underline{a}$, $\underline{b}$ and $\underline{c}$ are non-zero and non-coplanar vectors.

a) If $\alpha\,\underline{a} + \beta\,\underline{b} + \gamma\,\underline{c} = 0$, then show that $\alpha = \beta = \gamma = 0$. [10%]

b) If $\alpha_1\,\underline{a} + \beta_1\,\underline{b} + \gamma_1\,\underline{c} = \alpha_2\,\underline{a} + \beta_2\,\underline{b} + \gamma_2\,\underline{c}$, then show that $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$ and $\gamma_1 = \gamma_2$. [15%]

IV. A particle moves along the curve $\underline{r} = 5t^2\underline{i} + (3t^2 - 4t)\underline{j} + (3t - 5)\,\underline{k}$, where $t > 0$ is time. Find the components of the velocity and acceleration at time $t = 1$ in the direction, $4\underline{i} - 3\underline{j} + 5\underline{k}$. [25%]

V. Two particles start to move simultaneously at the same time from points $A = (1, 3)$ and $B = (-3, 2)$ with constant velocities $(\underline{i} + 3\underline{j})$ and $(\underline{i} + 4\underline{j})$ respectively. What is the time and distance apart when they are closest? [20%]

Q4.

I. Let $k, a \in \mathbb{R}$ such that, $k = \frac{a+9i}{1+ai}$. Determine the value of a. [10%]

II. Draw the locus of $arg\left(\frac{3-5z}{\sqrt{3}-i}\right) = \frac{\pi}{2}$. [20%]

III. Using the algebraic method, find the locus of $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$. [15%]

IV. Express $\ln(\sqrt{3} + i)$ in the form $x + iy$, where x and y are real numbers. [15%]

V. Using the De Moivre's theorem, [25%]

a) prove that $128cos^8\theta - cos8\theta = 4(14cos2\theta + 7cos4\theta + 2cos6\theta) + 35$.

b) find the values of x and y when the complex number: $(-1 + i)^7 = x + iy$, where x and y are real numbers. .

VI. Prove that $(1 - \omega + \omega^2)^8 + (\omega + \omega^2)^4 - 8\,(1 + \omega - \omega^2)^5 = -255$, where ω is a complex cubic root of unity. [15%]

SECTION – C

Q5.

I. Use the bisection method to find the root of the function $f(x) = x^2 - 3$ up to 4 decimal places in the interval $[1, 2]$. **[30%]**

II. With the initial guess $x_0 = 1.5$, Use the Newton - Raphson method to find the root of the $f(x) = x^3 - x - 1$, correct to five decimal places. **[30%]**

III. Apply the Gauss-Seidel method to solve the following system. **[40%]**

$$\begin{aligned} 4x - y - z &= 3 \\ -2x + 6y + z &= 9 \\ -x + y + 7z &= -6 . \end{aligned}$$

You may use the initial guess as $x = y = z = 0$.
(*Continue iterations until five successive approximations are identical when rounded to three significant digits*)

Q6.

I. Using the Lagrange's interpolation ,and the following data, estimate the number of workers who getting income not exceeding Rs. 2600 per day. **[25%]**

Income not exceeding (Rs.)	1500	2500	3000	3500
Number of workers	36	40	45	38

II. Estimate $f(1.83)$ using Newton's forward difference interpolation and the given data. **[40%]**

i	0	1	2	3	4
x_i	1.0	3.0	5.0	7.0	9.0
$f(x_i)$	0	1.0986	1.6094	1.9459	2.1972

III. Use Trapezoidal and Simpson's rules to evaluate $\int_0^1 \frac{1}{1+x^2}\, dx$, Hence find the value of π for each rule.*(Hint: Use 6 subintervals for the calculation.)*

[35%]

SECTION – D

Q7.

I. Define the Taylor series expansion of $f(x)$ about $x = a$. **[05%]**

a) Find the nth order Taylor series approximation of $f(x) = \frac{1}{(1+x)}$ about $x = 0$.
Hence derive nth order the Taylor series approximation of $(1 - x)^{-1}$ about $x = 0$. **[25%]**

b) Find the 4th order Taylor series approximation of the following function about $x = 0$. **[25%]**

$$g(x) = \frac{2x+1}{(x-5)(x+2)}.$$

(***Hint:*** *first find the partial function decomposition of the function and use the above part (a))*

II. Define the Laplace transform for a function $f(t), 0 \leq t < \infty$. **[05%]**

a) Find the Laplace transformation of the following function **[20%]**
$h(t) = t \sin 4t + 3e^{2t}(1 + \cos 3t) - \frac{e^{3t}}{t}$

b) Use Laplace transform to solve the differential equation $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 3y = 0$ with the initial conditions $y(0) = 2$ and $y'(0) = -1$. **[20%]**

Q8.

I. Let $f(x)$ be a periodic that is Riemann integrable function on the interval $(-l, l)$. Write down the Fourier series expansion for the function $f(x)$. **[10%]**

II. Let $g(x)$ be a function defined on the interval $(-\pi, \pi)$ by

$$g(x) = \begin{cases} -1, & -\pi < x < -\frac{\pi}{2} \\ 0, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} < x < \pi \end{cases}.$$

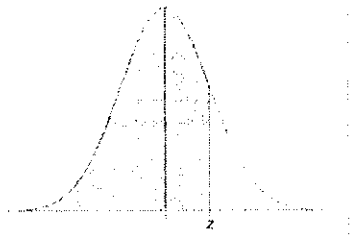
Find the Fourier series of $g(x)$. **[35%]**

III. Find the Fourier series corresponding to the function $h(x) = x^2$, for $0 < x < 2\pi$.

Hence, deduce that $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{\pi^2}{6}$. **[55%]**

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Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998