THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering

302

Final Examination (2021/2022)
MHZ5554: Engineering Mathematics IV

Date: 27th February 2023 (Monday) **Time: 1330 hrs. – 1630 hrs.**

Instructions:

- This paper consists of TWO (02) Parts: PART A and PART B.
- PART A is Compulsory and consists with TEN (10) short-answered questions.
- PART B consists with SEVEN (07) questions.
  - Answer any five (05) questions only. All questions carry equal marks.
  - Show all your workings.
  - Answer for each question should commence from a new page.
- State any assumptions you required.
- Number of pages in the paper is Five (05).
- All the symbols are in standard notation unless they are defined.
- All the relevant statistical tables are attached with this paper.
- This is a closed book test.
- Do not use red color pen.

PART A

1. Find the area between the parabolas $y^2 = x$ and $x^2 = y$.

(Marks 10)

2. Determine whether the complex series $\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{i}{3^n}\right)$ is convergent or not.

(Marks 10)

3. Write down the Maclaurin expansion of the complex function $f(z) = \frac{1}{1-z^2}$.

(Marks 10)

4. Find the integral $\int_C \frac{z^2+2z+2}{z^2-1}dz$, where C is the circle $|z| = \frac{1}{2}$.

(Marks 10)

5. Let the coordinate system $OX_1X_2X_3$ rotates about the OX_1 axis by an angle $\frac{\pi}{6}$ in the anti-clockwise sense. Then write down the transformation matrix.

(Marks 10)

6. Find the Fourier sine transformation of,

$$f(t) = \begin{cases} 1 & ; 0 < t < 1 \\ 0 & ; t > 1 \end{cases}$$

(Marks 10)

7. Find the minimum value of $Z = x + y$ subjected to constraints $-4x + y \geq 4$, $x, y \geq 0$.

(Marks 10)

8. If an experiment observed a p-value for an ANOVA as 0.00778, what you can say about the conclusion regarding the null hypothesis associated with this ANOVA at 5% significant level. *(Marks 10)*
9. Explain what it means by Degeneracy in solution of a linear programming problem. *(Marks 10)*
10. In an ANOVA it was found that the null hypothesis of a test was accepted at 5% significant level. If the same ANOVA is used to test the same hypothesis at 1% significant level Explain about the result of the test. *(Marks 10)*

PART B

Q1.

a. The Jacobian matrix J of transformation between two n – Dimensional spaces whose coordinates are labeled as u^i and v^i where $i = 1,2 \ldots, n$ is given by,

$$J = \begin{pmatrix} \frac{\partial u^1}{\partial v^1} & \cdots & \frac{\partial u^1}{\partial v^n} \\ \vdots & \ddots & \vdots \\ \frac{\partial u^n}{\partial v^1} & \cdots & \frac{\partial u^n}{\partial v^n} \end{pmatrix}$$

Using the transformation equations between the Cartesian and Cylindrical coordinate systems, find the Jacobian matrix J of the transformation between Cartesian to Cylindrical and Cylindrical to Cartesian coordinate systems.

(Marks 40)

b. Verify the Gauss divergence theorem for

$$\boldsymbol{F} = (x^2 - yz)\boldsymbol{i} + (y^2 - xz)\boldsymbol{j} + (z^2 - xy)\boldsymbol{k}$$

Taken over the rectangular parallelopiped bounded by the planes $x = 0, x = 1, y = 0, y = 2, z = 0$ and $z = 3$. *(Marks 60)*

Q2.

a. Verify Stokes' theorem for a vector field is defined by,

$$\boldsymbol{F} = (x^2 - y^2)\boldsymbol{i} + 2xy\boldsymbol{j}$$

over a rectangular surface in xy plane bounded by lines $x = 0, x = a, y = 0$ and $y = a$ where a is a non-zero real number and the boundary is described by counterclockwise sense.

(Marks 60)

b. Find the constants a, b and c such that,

$$\boldsymbol{F} = (x + 2y + az)\boldsymbol{i} + (bx - 3y - z)\boldsymbol{j} + (4x + cy + 2z)\boldsymbol{k}$$

is a conservative vector field and hence find $\phi(x, y, z)$ such that $\boldsymbol{F} = \nabla\phi$.

(Marks 40)

Q3.

a. For the conformal transformation $w = z^2$, show that the circle $|z - 1| = 1$ transforms into the cardioid $R = 2(1 + cos\phi)$, where $Re^{i\phi}$ in the $w -$ plane.

(Marks 30)

b. Find the Laurent series expansions of the function,

$$f(z) = \frac{1}{(z-1)(z-2)}\ ; z \in \mathbb{C} - \{1,2\}$$

from $z_0 = 0$ in each of the following domains where such an expansion exists.

i. $|z| < 1$

ii. $1 < |z| < 2$

iii. $|z| > 2$

(Marks 45)

c. Apply the Cauchy' s Residue Theorem to evaluate,

$$\int_C \frac{dz}{\sin(iz)},\ C: |z| = 4$$

(Hint: You may use the result "If $f(z) = \frac{\phi(z)}{\psi(z)}, \psi(a) = 0$ *but* $\phi(a) \neq 0$, *then* $Res(at\ z = a) = \frac{\phi(a)}{\psi'(a)}$ *")*

(Marks 25)

Q4.

a. Let $\boldsymbol{a}$ and $\boldsymbol{b}$ are two arbitrary vectors. Hence show that,

i. $\boldsymbol{a} \cdot \boldsymbol{b}$ is a zeroth order tensor. *(Marks 15)*

ii. $\boldsymbol{a} \times \boldsymbol{b}$ is a first order tensor. *(Marks 15)*

b. A set of Cartesian axes is rotated about the origin to coincide with the unit vectors, $\boldsymbol{e_1} = \left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right), \boldsymbol{e_2} = \left(\frac{3}{4}, \frac{\sqrt{3}}{4}, -\frac{1}{2}\right), \boldsymbol{e_3} = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$. Write down the rotation matrix corresponding to this rotation. *(Marks 25)*

c. Let the moment of inertia tensor $I_{i,j}$ is given by,

$$I_{i,j} = \begin{bmatrix} a & b & 0 \\ b & a & 0 \\ 0 & 0 & c \end{bmatrix}; \quad a, b, c \in \mathbb{R}$$

Hence find,

i. Eigen values and eigenvector matrix of $I_{i,j}$. *(Marks 25)*

ii. Matrix of tensor components referred to the principal axes. *(Marks 10)*

iii. Verify the result in the above part (ii), by using the Transformation Law of tensors. *(Marks 10)*

Q5.

a. Use the method of separation of variables to solve the partial differential equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

Given that $u(t,0) = u(t,1) = 0$ and $u = \sum_{n=1}^{\infty} \sin(n\pi x)$ when $t \to 0$.

(Marks 50)

b. Using the cosine Fourier Transform, solve the partial differential equation,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 \leq x < \infty, \qquad t > 0$$

Subject to the conditions,

i. $u(x,0) = 0 \ for \ x \geq 0.$ ii. $\frac{\partial u}{\partial x}(0,t) = -1$

(Marks 50)

Q6. Consider the following linear programming problem ,

$$Max\ Z = 500x + 300y$$

Subject to

$$15x + 5y \leq 300$$
$$10x + 6y \leq 240$$
$$8x + 12y \leq 450$$
$$x, y \geq 0$$

a.

i. Use the graphical method to solve the above linear programming problem. *(Marks 40)*

ii. What you can say about the optimal solution of the above linear programming problem. *(Marks 10)*

b.

i. Use simplex method to solve the above problem. *(Marks 40)*

ii. Confirm the comment that you have given about the optimal solution in part a (ii) using simplex method. *(Marks 10)*

Q7. Concrete is highly alkaline material that contains a higher pH value. In an experiment of three new concreate mixtures were tested. The pH value of concreate was recorded from three sample of three varieties of concreate and the recorded pH values are given in following table. The experimenter claims that the three varieties of concreate are having different pH level.

Variety 1	Variety 2	Variety 3
12.8	11.5	12.8
13.1	11.4	12.6
12.7	11.6	12.7
12.4	11.7	12.8
12.6		12.9
		12.7

a. Identify the factor variable and dependent variable. *(Marks 20)*

b. State the null and alternative hypothesis associated with the customers claim. *(Marks 20)*

c. Test the hypothesis about the customers claim at 5% significant level using ANOVA table and interpret the findings. *(Marks 50)*

d. State the assumptions you made for this test. *(Marks 10)*

End.

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(Marks 60)

b. Find the constants a, b and c such that,

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c. Apply the Cauchy' s Residue Theorem to evaluate,

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F Distribution: Critical Values of F (5% significance level)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.89	1.85	1.81	1.78
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.86	1.82	1.78	1.75
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.84	1.79	1.75	1.72
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.82	1.77	1.73	1.70
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.80	1.76	1.72	1.69
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.79	1.75	1.71	1.68
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.78	1.73	1.69	1.66
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.76	1.71	1.67	1.64
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.74	1.69	1.66	1.62
250	3.88	3.03	2.64	2.41	2.25	2.13	2.05	1.98	1.92	1.87	1.79	1.73	1.68	1.65	1.61
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.78	1.72	1.68	1.64	1.61
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85	1.78	1.72	1.67	1.63	1.60
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.77	1.71	1.66	1.62	1.59
600	3.86	3.01	2.62	2.39	2.23	2.11	2.02	1.95	1.90	1.85	1.77	1.71	1.66	1.62	1.59
750	3.85	3.01	2.62	2.38	2.23	2.11	2.02	1.95	1.89	1.84	1.77	1.70	1.66	1.62	1.58
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.76	1.70	1.65	1.61	1.58

F Distribution: Critical Values of F (5% significance level)

ν_1 / ν_2	25	30	35	40	50	60	75	100	150	200
1	249.26	250.10	250.69	251.14	251.77	252.20	252.62	253.04	253.46	253.68
2	19.46	19.46	19.47	19.47	19.48	19.48	19.48	19.49	19.49	19.49
3	8.63	8.62	8.60	8.59	8.58	8.57	8.56	8.55	8.54	8.54
4	5.77	5.75	5.73	5.72	5.70	5.69	5.68	5.66	5.65	5.65
5	4.52	4.50	4.48	4.46	4.44	4.43	4.42	4.41	4.39	4.39
6	3.83	3.81	3.79	3.77	3.75	3.74	3.73	3.71	3.70	3.69
7	3.40	3.38	3.36	3.34	3.32	3.30	3.29	3.27	3.26	3.25
8	3.11	3.08	3.06	3.04	3.02	3.01	2.99	2.97	2.96	2.95
9	2.89	2.86	2.84	2.83	2.80	2.79	2.77	2.76	2.74	2.73
10	2.73	2.70	2.68	2.66	2.64	2.62	2.60	2.59	2.57	2.56
11	2.60	2.57	2.55	2.53	2.51	2.49	2.47	2.46	2.44	2.43
12	2.50	2.47	2.44	2.43	2.40	2.38	2.37	2.35	2.33	2.32
13	2.41	2.38	2.36	2.34	2.31	2.30	2.28	2.26	2.24	2.23
14	2.34	2.31	2.28	2.27	2.24	2.22	2.21	2.19	2.17	2.16
15	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.10
16	2.23	2.19	2.17	2.15	2.12	2.11	2.09	2.07	2.05	2.04
17	2.18	2.15	2.12	2.10	2.08	2.06	2.04	2.02	2.00	1.99
18	2.14	2.11	2.08	2.06	2.04	2.02	2.00	1.98	1.96	1.95
19	2.11	2.07	2.05	2.03	2.00	1.98	1.96	1.94	1.92	1.91
20	2.07	2.04	2.01	1.99	1.97	1.95	1.93	1.91	1.89	1.88
21	2.05	2.01	1.98	1.96	1.94	1.92	1.90	1.88	1.86	1.84
22	2.02	1.98	1.96	1.94	1.91	1.89	1.87	1.85	1.83	1.82
23	2.00	1.96	1.93	1.91	1.88	1.86	1.84	1.82	1.80	1.79
24	1.97	1.94	1.91	1.89	1.86	1.84	1.82	1.80	1.78	1.77
25	1.96	1.92	1.89	1.87	1.84	1.82	1.80	1.78	1.76	1.75
26	1.94	1.90	1.87	1.85	1.82	1.80	1.78	1.76	1.74	1.73
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29	1.89	1.85	1.83	1.81	1.77	1.75	1.73	1.71	1.69	1.67
30	1.88	1.84	1.81	1.79	1.76	1.74	1.72	1.70	1.67	1.66
35	1.82	1.79	1.76	1.74	1.70	1.68	1.66	1.63	1.61	1.60
40	1.78	1.74	1.72	1.69	1.66	1.64	1.61	1.59	1.56	1.55
50	1.73	1.69	1.66	1.63	1.60	1.58	1.55	1.52	1.50	1.48
60	1.69	1.65	1.62	1.59	1.56	1.53	1.51	1.48	1.45	1.44
70	1.66	1.62	1.59	1.57	1.53	1.50	1.48	1.45	1.42	1.40
80	1.64	1.60	1.57	1.54	1.51	1.48	1.45	1.43	1.39	1.38
90	1.63	1.59	1.55	1.53	1.49	1.46	1.44	1.41	1.38	1.36
100	1.62	1.57	1.54	1.52	1.48	1.45	1.42	1.39	1.36	1.34
120	1.60	1.55	1.52	1.50	1.46	1.43	1.40	1.37	1.33	1.32
150	1.58	1.54	1.50	1.48	1.44	1.41	1.38	1.34	1.31	1.29
200	1.56	1.52	1.48	1.46	1.41	1.39	1.35	1.32	1.28	1.26
250	1.55	1.50	1.47	1.44	1.40	1.37	1.34	1.31	1.27	1.25
300	1.54	1.50	1.46	1.43	1.39	1.36	1.33	1.30	1.26	1.23
400	1.53	1.49	1.45	1.42	1.38	1.35	1.32	1.28	1.24	1.22
500	1.53	1.48	1.45	1.42	1.38	1.35	1.31	1.28	1.23	1.21
600	1.52	1.48	1.44	1.41	1.37	1.34	1.31	1.27	1.23	1.20
750	1.52	1.47	1.44	1.41	1.37	1.34	1.30	1.26	1.22	1.20
1000	1.52	1.47	1.43	1.41	1.36	1.33	1.30	1.26	1.22	1.19