00159

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical and Computer Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: **EEX6441 Electromagnetism and Wave Propagation**
Academic Year	: 2021/2022
Date	: 13th Monday, February 2023
Time	: 1330 - 1630 hrs

General Instructions

1. Read all the instructions carefully before answering the questions.
2. This is a Closed Book Test (CBT).
3. This question paper consists of **Six (06)** questions in **Three (03)** pages.
4. Answer **only five (05) questions** by answering **ALL in Section A** and selecting only **two (02) from Section B**.
5. All questions carry equal marks.
6. The answer for each question should commence from a new page.
7. Answers should be in clear handwriting.
8. Do not use a Red color pen.
9. All the notations have their usual meaning.
10. Assume any missing parameters with suitable values.

Section A

Answer all questions in this section.

Q1.

(a) State the Coulomb's law. [2 Marks]

(b) Define the curl of a vector and explain its physical significance. [4 Marks]

(c) Derive equations for the capacitance of two scenarios shown in the Figure Q1 and compare them. [6 Marks]

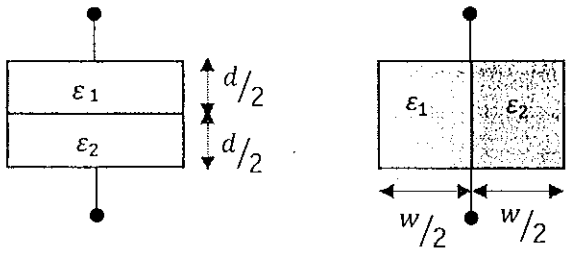


Figure Q1

(d) Two point charges having equal mass m and charge Q are suspended at a common point by two threads of negligible mass and a length l. Derive an expression for the inclination angle of each thread to the vertical.

. [8 Marks]

Q2.

(a) Given that $\mathbf{D} = z\rho\cos^2\phi\,\mathbf{a}_z\ {}^{C}/_{m^2}$, calculate the charge density at $(1, \frac{\pi}{4}, 3)$ and the total charge enclosed by the cylinder having radius 1 m with $-2\text{ m} \leq z \leq 2\text{ m}$.

[10 Marks]

(b) Two conducting cones at $\theta = \frac{\pi}{10}$ and $\theta = \frac{\pi}{6}$ of infinite extent are separated by a tiny gap at $r = 0$. If $V\left(\theta = \frac{\pi}{10}\right) = 0$ and $V\left(\theta = \frac{\pi}{6}\right) = V_0$, find V and $\mathbf{E}$ between the cones.

[10 Marks]

Q3.

(a) State the Ampere's law. [2 Marks]

(b) Explain the magnetic scalar potential and the magnetic vector potential.

[4 Marks]

(c) Using the Ampere's circuital law, determine the magnetic flux between the conductors of a coaxial cable having inner and outer radii r_1 and $r_2 (> r_1)$, a length l and an enclosed current of I. [7 Marks]

(d) An infinitely long conductor carries a current I in the direction of z −axis. Determine the magnetic field intensity $\mathbf{H}$ at a point P using the cylindrical coordinate system. [7 Marks]

Section B

Select only two (02) questions from this section.

Q4.

(a) What is an electromagnetic wave? Explain. [2 Marks]

(b) What is meant by

(i) propagation constant?

(ii) attenuation constant?

(iii) phase constant? [6 Marks]

(c) The electric field intensity of an electromagnetic wave in free space is given by $\mathbf{E} = E_0 \sin(\omega t - \beta y)\mathbf{a}_y$ V/m. Calculate

(i) electric flux density $\mathbf{D}$,

(ii) magnetic flux density $\mathbf{B}$ and

(iii) magnetic field intensity $\mathbf{H}$. [12 Marks]

Q5.

(a) Explain the following.

(i) Linear Polarization

(ii) Circular Polarization

(iii) Elliptical Polarization [3 Marks]

(b) Discuss the reflection of a wave at a perfect dielectric in case of oblique incident. [5 Marks]

(c) In free space, a plane wave with $\mathbf{H} = 10\cos(10^8 t - \beta z)\, a_x$ $^{mA}/_{m}$ is incident on a lossless medium $\varepsilon = 2\mu_0, \mu = 8\mu_0$ in the region $z \geq 0$. Determine the reflected wave $\mathbf{H}_r, \mathbf{E}_r$ and the transmitted wave $\mathbf{H}_t, \mathbf{E}_t$. [12 Marks]

Q6.

(a) Explain the significance of cut-off frequency in waveguides. [3 Marks]

(b) Explain the three different modes in waveguides. [4 Marks]

(c) A rectangular waveguide having dimensions $a = 2.5$ cm, $b = 1$ cm, is to be operated below 15.1 GHz. How many TE and TM modes can the waveguide transmit, if it is filled with a medium characterized by $\alpha = 0$, $\varepsilon = 4\varepsilon_0$ and $\mu_r = 1$? Calculate the cut-off frequencies of those modes. [13 Marks]

Note:

Cylindrical Coordinates

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{\mathbf{z}}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}},$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta A_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi},$$

$$\begin{aligned}\nabla \times \mathbf{A} = {} & \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}\left(A_\varphi \sin\theta\right) - \frac{\partial A_\theta}{\partial \varphi}\right)\hat{\mathbf{r}} \\ & + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}\left(rA_\varphi\right)\right)\hat{\boldsymbol{\theta}} \\ & + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rA_\theta\right) - \frac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol{\varphi}},\end{aligned}$$

$$\begin{aligned}\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2} \\ &= \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right)f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \varphi^2}f.\end{aligned}$$