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The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical & Computer Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: EEX7339 Information Theory & Coding**
Academic Year	: 2021/2022
Date	: 3rd March 2023
Time	: 1400-1700hrs
Duration	: **3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Five (5)** questions in **Three (3)** pages.
3. Answer **All** the questions.
4. Answer for each question should commence from a new page.
5. Clearly state all your assumptions.
6. This is a Closed Book Test **(CBT).**
7. Answers should be in clear handwriting.
8. Do not use red colour pens.

Q1.

(a) Let a pair of mutually dependent tetrahedron non-fair dices (a solid shape having four equilateral triangle faces) is tossed simultaneously. Let us denote the outcome from the dice toss as $\{D_1, D_2\}$. Two variable probability distribution is given by,

		D_1			
	$p(D_1,D_2)$	1	2	3	4
D_2	1	1/16	1/16	0	1/8
	2	0	1/16	1/8	1/16
	3	1/8	0	1/16	1/16
	4	1/16	1/8	1/16	0

Find,

i. $H(D_1)$ and $H(D_2)$ **(4 Marks)**
ii. $H(D_1|D_2)$ and $H(D_2|D_1)$ **(4 Marks)**
iii. $H(D_1, D_2)$ **(4 Marks)**
iv. $H(D_2) - H(D_2|D_1)$ **(2 Marks)**
v. $I(D_1; D_2)$ **(2 Marks)**

(b) For three general outcomes X, Y and Z, prove the inequality $I(X; Z|Y) \geq I(Z; Y|X) - I(Z; Y) + I(X; Z)$. **(4 Marks)**

Q2.

(a) Define mutual information $I(x; y)$. **(2 Marks)**

(b) How does mutual information between the input and output signals in a communication channel relate to the capacity of the channel? **(4 Marks)**

(c) The binary symmetric channel (BSC) is frequently used to model practical communication systems. Let the channel matrix of a certain BSC is given by,

$$\begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix}$$

i. Explain the elements of this matrix. **(4 Marks)**
ii. Find an expression for the average mutual information in this BSC. **(5 Marks)**
iii. Hence find the capacity of this BSC. **(5 Marks)**

Q3.

(a) Clearly explain the reason for the inefficiency in fixed length codes for alphabets whose letters are non-equiprobable. **(2 Marks)**

(b) Let a certain memoryless source emits five different symbols $x_i, i \in \{1, 2, 3, 4, 5\}$ with the probabilities $P(x_i) = \lambda e^i$.

i. Find the value of the constant λ. **(1 Marks)**

ii. Find the entropy of the source. **(2 Marks)**

iii. Find a codeword set having minimum possible equal length codewords to represent these symbols, without loss of any information. **(2 Marks)**

iv. Use Huffman's technique to find a uniquely decodable prefix codeword set for this symbol set. **(5 Marks)**

v. Find the average codeword length for the derived codeword set in iv. **(2 Marks)**

vi. Calculate the code rate for the codeword set in iv. **(3 Marks)**

vii. Is this code in iv optimal? Reason your answer. **(3 Marks)**

Q4.

(a) List a property of a linear block channel code. **(1 Marks)**

(b) Consider a $(7, 4)$ Hamming code with the generator matrix,

$$G = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Find,

i. The complete codeword set. **(4 Marks)**

ii. minimum Hamming distance of this code. **(2 Marks)**

iii. error correcting capability of the code. **(1 Marks)**

iv. error detection capability of the code. **(1 Marks)**

v. parity check matrix, H. **(4 Marks)**

vi. syndrome vector for a received bit vector (noise corrupted) 1101101. **(2 Marks)**

vii. the correct codeword using maximum-likelihood decoder corresponding to a received bit vector 1101101. **(5 Marks)**

Q5. Consider a rate 1/3 convolution code given by the following generator sequences.

$$g_1(x) = 011,\ g_2(x) = 110,\ g_3(x) = 111$$

(a) draw the convolutional encoder. **(2 Marks)**

(b) derive the state diagram. **(3 Marks)**

(c) derive the trellis diagram. **(5 Marks)**

(d) considering that the rightmost bit enters the decoder first find the encoder output for an input sequence of 10100. **(5 Marks)**

(e) Use Viterbi algorithm to find the decoded output for a bit sequence 100 011 100 100 011 at the decoder input. **(5 Marks)**