The Open University of Sri Lanka

B.Sc/B.Ed Degree Programme

No Book Test (NBT) – 2024/2025

Pure Mathematics – Level 05

PEU5304 – Introduction to Complex Analysis

Duration: - One Hour.

Date: 29.09.2024 **Time: 10.30 AM – 11.30 AM**

Answer All Questions.

1. (a). Find the polar form of each of the following complex numbers $(\sqrt{3}-i)$ and $(1-\sqrt{3}i)$.

   Hence, express $(\sqrt{3}-i)(1-\sqrt{3}i)$ in the polar form.

   (b). State the De Moivre's Theorem. By using this theorem,

   (i). show that $\cos^3\theta = \frac{\cos 3\theta + 3\cos\theta}{4}$,

   (ii). express $(1+i)^{24}$ in the cartesian form.

2. (a). Write down the n distinct roots of $[r_0(\cos\theta_0 + i\sin\theta_0)]^{\frac{1}{n}}$, where n is a positive integer and $r_0 > 0$.

   Hence, evaluate $(27i)^{\frac{1}{3}}$.

   (b). Let $p(z) = 2z^3 + 2iz^2 - (10+5i)z + 14i - 5 = 0$ be a complex cubic equation.

   (i). Verify that i is a root of $p(z) = 0$.

   (ii). Find all the other roots of $p(z) = 0$.

   (c). It is given that $z = e^{i\theta}$, $\theta \in \mathbb{R}$. Show that $z + {}^1/_z = 2\cos\theta$.

   Hence, show that, $z^2 - z + 2 - {}^1/_z + {}^1/_{z^2} = 4\cos^2\theta - 2\cos\theta$.