THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering
Bachelor of Software Engineering

005

Final Examination (2021/2022)
MHZ4256: Mathematics for Computing

Date: 01st March 2023 (Wednesday) **Time: 0930 hrs. – 1230 hrs.**

Instructions:

- This paper consists of Nine (9) questions in Six (06) pages.
- Choosing at least one from each single section, answer a total of six (6) questions only.
- State any assumptions you require.

Section A

Q1.

I. Write the truth values of the following statements.

a. 9 is a perfect square but 11 is a prime number. (10%)

b. Neither 21 is a prime number nor it is divisible by 3. (10%)

II. State the converse, inverse, and contrapositive of the following conditional statements.

a. If a function is differentiable, then it is continuous. (15%)

b. If it rains, then the match will be cancelled. (15%)

III. Using the truth tables, prove the following logical equivalences.

a. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ (15%)

b. $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$ (15%)

IV. Write a negation of the following statements.

a. $\forall n \in \mathbb{N},\ n + 7 > 6.$ (10%)

b. If the weather is fine, then my friend will come, and we will go for a picnic. (10%)

Q2.

I. Using laws of propositional logic, prove that,

$$\big((p \rightarrow q) \wedge \neg q\big) \rightarrow \neg p$$

is tautology. (30%)

II. Test the validity of the following argument without using a truth table.

"If the grass was wet, then it must have rained last night, or the sprinkler was on. It did not rain last night, but the grass was wet. Therefore, the sprinkler was on."

III. Check the validity of the following argument. (35%)

"Everyone who is running has strong legs. All those who do not run are unhealthy. Nimal does not have strong legs. Therefore, Nimal is unhealthy." (35%)

Q3.

I. Using the principle of mathematical induction, prove that $n^3 + 5n$ is divisible by 3 for all $n \in \mathbb{N}$. (25%)

II. Use the method of contraposition to prove the statement "for any integers a and b, if $a + b \geq 15$, then $a \geq 8$ or $b \geq 8$." (25%)

III. Prove that "there is no largest even integer". (25%)

IV. Let n be an integer. Use a proof by cases to show that $2n^2 + n + 1$ is not divisible by 3. (25%)

Section B

Q4.

I. State which of the following statements are true and which are false. Justify your answer.

a. $3 \notin \{x \in \mathbb{C} | x^4 - 5x^3 + 12x^2 - 112x + 6 = 0\}$ (15%)

b. Let R and S are two sets defined as follows.

$$R = \{x \in \mathbb{Z} \mid x \text{ is divisible by } 2\}$$

$$S = \{x \in \mathbb{Z} \mid x \text{ is divisible by } 3\}$$

$$R \cap S = \emptyset$$

(15%)

II. Without using Venn diagrams, prove the following:

a. $(A - B) \cap (C - B) = (A \cap C) - B$ (20%)

b. $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = B \cap C'$ (20%)

III. Two finite sets have m and n elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the values of m and n. (30%)

Q5.

I. Let $A = \{1,2,3\}$ and $B = \{4,5,7\}$. Find,

a. $A \times A$ and $B \times B$. (15%)

b. Let $R = \{A \times A\} \cup \{B \times B\}$. State whether R is an equivalence relation or not. Justify your answer. (15%)

II. Show that the relation R is defined on the set of integers by xRy if and only if $2|(x + y)$. (35%)

III. Show that the relation " $\subseteq$ " of a set of inclusion is a partial order relation. (35%)

Q6.

I. Give diagrammatic example of a function $f: A \to B$ where $A = \{a, b, c\}$ and $B = \{d, e, f\}$, which is,

a. onto but not one to one. (10%)

b. neither one to one nor onto. (10%)

c. both one to one and onto. (10%)

II. Let $f: A \to \mathbb{R}$ be a function is defined by,

$$f(x) = 5 - \frac{1}{1+x}$$

a. Find the domain of f. (10%)

b. Show that the function f is one to one. (10%)

c. Find the inverse of f and hence find the range of f. (20%)

III. Let $S = \{1,2,3,4,5\}$ and let $f, g, h: S \to S$ be the functions defined by,

$$f = \{(1,2), (2,1), (3,4), (4,5), (5,3)\}$$
$$g = \{(1,3), (2,5), (3,1), (4,2), (5,4)\}$$
$$h = \{(1,2), (2,2), (3,4), (4,3), (5,1)\}$$

a. Explain why f and g have inverses but h does not. Find f^{-1} and g^{-1}. (15%)

b. Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1} \neq f^{-1} \circ g^{-1}$. (15%)

Section C

Q7.

I. Consider the following truth table.

A	B	C	F
0	0	0	1
0	1	1	0
0	1	0	0
0	0	1	1
1	1	0	1
1	0	0	0
1	0	1	0
1	1	1	1

where A, B and C are input variables and F is an output variable.

a. Create a Karnaugh map (K-map) based on the above truth table. (10%)

b. Using the K-map created in the above part (a)-(i), write down the minimized Boolean function. (20%)

II. Consider the following Boolean function given below.

$$(\bar{A}B + AB)(\bar{C}D + CD) + (\bar{A}\bar{B} + AB + \bar{A}B + A\bar{B})(\bar{C}D) + (\bar{A}\bar{B} + A\bar{B})(\bar{C}\bar{D} + C\bar{D})$$

Create a K-map based on the above Boolean function and minimize the above Boolean function using the created K-map. (30%)

III. Minimize the following Boolean functions using algebraic method.

a. $(A + \bar{A})(\bar{B}\bar{C} + \bar{B}C) + A(\bar{B}\bar{C} + \bar{B}C + B\bar{C} + BC) + (A + \bar{A})(\bar{B}\bar{C} + B\bar{C})$ (20%)

b. $(AB + A\bar{B})(C\bar{D}) + (\bar{A}\bar{B} + A\bar{B})(\bar{C}\bar{D} + C\bar{D})$ (20%)

Q8.

I. Find the smallest positive integer n for which, $\left(\frac{1+i}{1-i}\right)^n = 1$. (20%)

II. If $a^2 + b^2 + c^2 = 1$ and $b + ic = (1 + a)z$, then prove that,

$$\frac{a + ib}{1 + c} = \frac{1 + iz}{1 - iz}; \quad a, b, c \in \mathbb{R} \text{ and } z \neq -i \tag{40\%}$$

III. Let $z, w \in \mathbb{C}$. Show that $|z|^2 = z\bar{z}$. Deduce that $|z - w|^2 = |z|^2 - 2\text{Re}(z\bar{w}) + |w|^2$. Write a similar expression for $|1 - z\bar{w}|^2$ and hence show that,

$$|z - w|^2 - |1 - z\bar{w}|^2 = -(1 - |z|^2)(1 - |w|^2) \tag{40\%}$$

Q9.

I. Sketch on the same Argand diagram, the loci of points representing complex numbers z satisfying,

a. $|z - i| = 1$ (15%)

b. $Arg(z - i) = \frac{\pi}{6}$ (15%)

and the complex number represented by the point of intersection of these loci in the form $r(\cos\theta + i\sin\theta)$ where $r > 0$ and $0 < \theta < \frac{\pi}{2}$. (25%)

II. Let consider the statement, If $AB : AC = a : b$ and θ is the angle measured from AB to AC in the anti-clockwise sense, then $z_C - z_A = \frac{b}{a}(z_B - z_A)(\cos\theta + i\sin\theta)$, where A, B and C be three distinct points which represent complex numbers z_A, z_B and z_C respectively in the argand diagram.

The points P, Q, R and S taken in the anti-clockwise sense, form a square in the argand diagram. Let $-4 - i$ and $2 - 3i$ be the complex numbers represented by the points P and Q respectively. Find the complex numbers represented by the points R and S using the above statement. (45%)

-End-

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