The Open University of Sri Lanka
B.Sc. Degree Programme, Level – 04
Final Examination – 2024/2025
PHU4303 – Mathematical Methods for Physics
Duration: 2 hours

Date: 07th December 2024 **Time: 9.30 a.m. to 11.30 a.m.**

This paper has 6 questions. **Answer any four (4) questions**
Non-programmable calculators are allowed.

1)

a) A student dissolve 10g of salt in 30 ml of water. Assume salt is completely dissolved, evenly distributed in water and no change of total volume. This solution is called solution 1.

i) He removes 1/3rd volume from the solution 1 and add water to it until the total volume reaches 30 ml. After mixing well, it is called solution 2. What is the total weight of salt in the solution 2 ? (02 marks)

ii) He removes 1/3rd volume from the solution 2 and add water to it until the total volume reaches 30 ml. After mixing well, it is called solution 3. What is the total weight of salt in the solution 3 ? (03 marks)

iii) Student repeats the above process until he creates solution 10. What is the total weight of salt dissolved in solution 10? (05marks)

b)

i) If $a = \frac{dv}{dt}$ solve the said differential equation to find the general solution for V. (05 marks)

ii) Initial condition is given as $v(0) = u$; Find the particular solution. (05 marks)

iii) Consider previous solution. If $v = \frac{ds}{dt}$ and $s(0) = 0$ find the particular solution for S. (05 marks)

2)

a)

The Hamiltonian matrix represents the total energy of a quantum system. The eigenvalues of The Hamiltonian matrix represent the possible energy levels of the system. The eigenvectors represent the quantum states associated with these energy levels.
Suppose a particle exists in a quantum state and the Hamiltonian of the system is:

$$H = \begin{pmatrix} 5 & 2 \\ 2 & 5 \end{pmatrix}$$

i) Find the possible Energy Levels. (10marks)

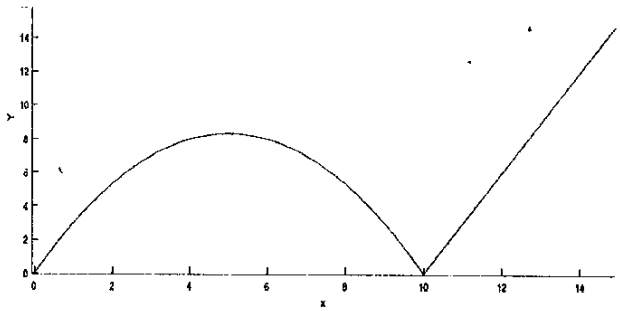
ii) quantum states associated with these energy levels. (10 marks)

b) Calculate $\frac{5-3i}{10+2i}$ where $i = \sqrt{-1}$. Clearly show the intermediate steps. (05 marks)

3)

Consider the following piecewise function.

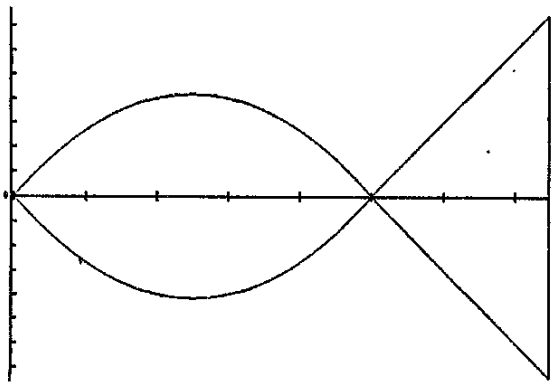
$$f(x) = \begin{cases} \dfrac{25-(x-5)^2}{3} \; ; 0 \le x < 10 \\ 3x - 30 \; ; 10 \le x < 15 \end{cases}$$



a) Use differentiation to find x, where f(x) is maximum, when x is between 0 and 10.
(10 marks)

b) Calculate the area covered by the f(x) and the x axis when x is between 0 and 15. (10 marks)

c) Let g(x) = -f(x). Calculate the area inside the shape of the fish created by plotting f(x) and g(x).
(05 marks)



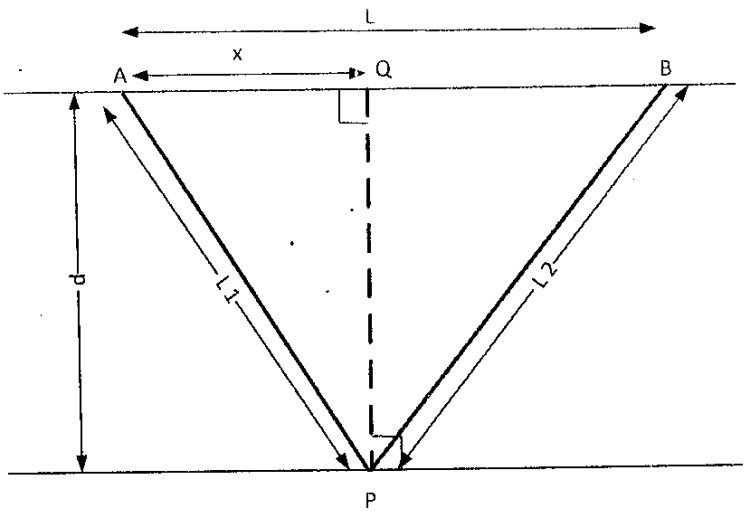
4)

a) Use bisection method to find the root of $f(x) = x^3 - x - 2$ in the interval [1,2]. Calculate 8 iterations and put the results in the table given below. Add more rows to following table as needed.
(15 marks)

Iteration	a	b	midpoint	f(midpoint)
1				
2				
....				
8				

b) Calculate the total derivative of the function $f(x, y, z) = x^2 \sin(x) + xyz + z\cos(x) - 10$
(10 marks)

5) Consider the following diagram. A beam of light is emitted from point A, It is reflects from a mirror at point P and reach point B. Distance between point A and B is 'L'. ,Distance between the AB plane and the mirror is 'd'.



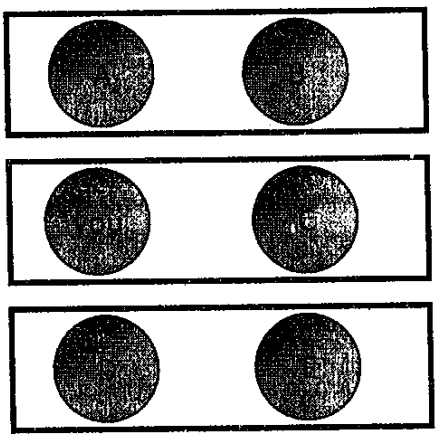
a) Obtain expression for the distances L1 and L2 using terms 'd' , 'L' , and 'x'. (8 marks)

b) Obtain an expression for the total distance traveled by the light beam when it travels from Point A to Point B by reflecting off point P. (7 marks)

c) Light in a homogeneous medium travel at constant speed. Hence shortest path is the fastest path. Using differentiation, find the value of x, which minimize the time taken by the light beam to travel from point A to B by reflecting on the mirror. (10 marks)

6)

a) You are sitting on a bus. There are five vacant seats around you. Two seats (A and B) Infront of you. One seat next to you (C). Two seats behind you (D and E). A person who has fever gets on to the bus. Assume there is an equal chance of the person to sit on any of the vacant seats. If the person is sitting next to you, there is a 1/2 chance you will catch the fever. If he is sitting Infront (A or B) that chance drops to 1/6. If the person is sitting behind you (D or E) the change is 1/5.



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i) Draw a tree diagram to show the probabilities (05 marks)

ii) Calculate the probability of you catching the fever (05 marks)

iii) If you catch the fever, what is the probability he had sat Infront of you (A or B) (05 marks)

b) Calculate the divergent of the following vector $\boldsymbol{V} = xy\boldsymbol{i} + \cos(x)\,\boldsymbol{j} - xz^2\boldsymbol{k}$ (10 marks)

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