00179

THE OPEN UNIVERSITY OF SRI LANKA

B.Sc./B.Ed DEGREE PROGRAMME – LEVEL 04

FINAL EXAMINATION -2024/2025

PURE MATHEMATICS

PEU4302/PEE4302 – Linear Algebra

DURATION: Two (02) Hours

Date: 21.11.2024 **Time:1.30 p.m.-3.30 p.m.**

Answer FOUR questions only.

1. (a) Define each of the following:

(i) Inverse of a matrix.

(ii) Orthogonal matrix.

(iii) Skew-symmetric matrix.

(iv) Equivalent matrices.

(b) Prove that $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$

(c) Let $A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{pmatrix}$. Find the matrix B where $AB = \begin{pmatrix} 3 & 4 & 2 \\ 1 & 6 & 1 \\ 5 & 6 & 4 \end{pmatrix}$.

(d) Find the inverse of the following matrix:

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{pmatrix}.$$

2. (a) Convert the following matrix to the triangular form and hence find the rank.

$$\begin{pmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & 4 & 7 \\ -1 & -3 & 2 & -1 \\ 2 & 4 & -1 & 3 \end{pmatrix}.$$

(b) Use Cayley - Hamilton theorem to determine A^{-1}, A^{-2} and A^{-3} if

$$A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}.$$

(c) Find B if $B = A^7 - 4A^6 - A^5 + 5A^4 - 4A^3 - 2A^2 + 5A + I$.

3. (a) Convert the following matrix to the row reduced echelon form

$$\begin{pmatrix} 1 & 2 & -3 & -4 & 6 \\ 1 & 3 & 1 & -2 & 4 \\ 2 & 5 & -2 & -5 & 10 \end{pmatrix}.$$

(b) Solve the following system of equations using part (i):

$$\begin{aligned} x_1 + 2x_2 - 3x_3 - 4x_4 &= 6 \\ x_1 + 3x_2 + x_3 - 2x_4 &= 4 \\ 2x_1 + 5x_2 - 2x_3 - 5x_4 &= 10 \end{aligned}$$

(c) Find for what values of α and β the system of linear equations

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \alpha z &= \beta \end{aligned}$$

has

(i) a unique solution,

(ii) no solution,

(iii) infinite number of solutions. Also find the solutions in the cases (i) and (iii).

4. (a) Determine α, β, γ so that A is orthogonal, where

$$A=\begin{pmatrix} \alpha & 2\beta & \gamma \\ 2\alpha & \beta & -\gamma \\ -2\alpha & 2\beta & -\frac{\gamma}{2} \end{pmatrix}.$$

(b) Transform the quadratic form $2x_1x_2+2x_1x_3-2x_2x_3$ to canonical form by orthogonal transformation. Write the orthogonal transformation which reduces the Quadratic form to Canonical form.

5. (a) Determine the algebraic and geometric multiplicity of $A=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{pmatrix}$.

(b) Solve the following system of equations by LU-decomposition:

$$4x_1 + 8x_2 + 4x_3 = 8$$
$$x_1 + 5x_2 + 4x_3 - 3x_4 = -4$$
$$x_1 + 4x_2 + 7x_3 + 2x_4 = 10$$
$$x_1 + 3x_2 - 2x_4 = -4.$$

6. (a) Show that $A=\begin{pmatrix} 2i & -2+3i & -4i \\ 2+3i & 5i & -6 \\ -4i & 6 & 3i \end{pmatrix}$ is skew-Hermitian.

(b) Find the skew-Hermitian form S for A with $X=\begin{pmatrix} 1 \\ 0 \\ i \end{pmatrix}$.

(c) Find eigen values and eigen vectors of $\begin{pmatrix} 9 & -1 \\ 5 & 7 \end{pmatrix}$.