The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination - 2024/2025
Pure Mathematics - Level 03
PEU3301 – Foundations of Mathematics

Duration: Two hours

Date: 28-04-2025 **Time: 01.30 p.m. – 3.30 p.m.**

General Instructions

1. Read all the instructions carefully before answering the questions.
2. This paper consists of **TWO** sections, Section **A** and Section **B**. Section **A** is compulsory and it consists of **SIX** Structured Essay Questions.
3. Section **B** consists of **FIVE** essay type questions and answers only **THREE** of them. Each question in Section **B** carries 100 marks.

SECTION A

Answer **ALL** the Questions.

1.

a) Prove that the set $A = \{n \in \mathbb{N}: n^2 + n + 23\}$ is non-empty.

b) Let A, B and C be sets. Show that $(A \cap B) \cap C = A \cap (B \cap C)$.

c) Find the range of the function $f(x) = \frac{3x-1}{x-1}, x \in \mathbb{R} \setminus \{1\}$.

d) Let $f(x) = \sqrt{x^2 - 4}$, $x \in [2, \infty)$ and $g(x) = \frac{x^2}{x^2+4}$, $x \in \mathbb{R}$. Find $g \circ f$.

e) Let $f(x) = x^2$, $x \in \mathbb{R}$. Find $f([-1,3])$.

f) Using axioms of addition. show that $(-0) = 0$.

SECTION B

Answer **THREE** Questions **ONLY**.

2.

a) Solve the inequality $\frac{|2x-3|}{3x-2} \leq 4$, where $x \in \mathbb{R}$.

b) Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x - 3$. Find $f^{-1}([2, 9))$.

c) Let x and y be integers such that $x + y$ is odd. Prove that x is even or y is even.

3.

a) Let R be the relation defined on the set $\mathbb{R}$ of all real numbers by xRy if $x \leq y$ for all $x, y \in \mathbb{R}$.

i. Show that R is a partial order relation on $\mathbb{R}$..

ii. Is the relation R a total order relation on $\mathbb{R}$? Justify your answer.

b) Let $m \in \mathbb{N}$. Using the *Principle of Mathematical Induction* prove that,

i. $m + n \in \mathbb{N}$ for every $n \in \mathbb{N}$.

ii. $mn \in \mathbb{N}$ for every $n \in \mathbb{N}$.

4.

a) Let $f(x) = \frac{2}{x}$, $x \in (0, 1)$.

i. Show that f is a bijection from $(0, 1)$ into $(1, \infty)$.

ii. Is the inverse function of f exists? If yes, find the inverse function $f^{-1}(x)$.

b) Let α be a non-zero algebraic number. Show that $\frac{1}{\alpha}$ is an algebraic number.

5.

a) Prove that each real number c is an upper bound of the empty set $\emptyset$.

b) Consider the open interval $(0, 2)$.

i. Show that $(0, 2)$ is bounded.

ii. Prove that the set $(0, 2)$ does not have a minimum.

iii. Show that $\sup(0, 2) = 2$.

6.

a) Let $a_n = \frac{1}{3^{2n-1}}$ for $n \in \mathbb{N}$.

   i. Find the nth partial sum $S_n = \sum_{k=1}^{n} a_k$, for $n \in \mathbb{N}$.

   ii. Show that the series $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} a_n = \frac{3}{4}$.

b) Let $b_n = \frac{1}{\sqrt{n}}$ for $n \in \mathbb{N}$. Show that the series $\sum_{n=1}^{\infty} b_n$ diverges.

c) By using the definition for limit of a sequence prove that $\lim_{n \to +\infty} \frac{1}{n} = 0$.

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