**The Open University of Sri Lanka**

**B.Sc/B.Ed. Degree Programme – Level 04**

**Open Book Test– 2024/2025**

**Pure Mathematics**

**PEU4303 – Group Theory 1**

**Duration: - One Hour.**

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**Date: -14 .02.2025** **Time: - 02.30 p.m. – 03.30 p.m.**

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**Answer All Questions.**

(01) An operation $*$ is defined on a set of numbers $S$ by $x * y = \frac{xy}{x+y}$.
Decide whether $*$ is a binary operation in each of the following cases, giving a specific Counter-example where appropriate.

(i) $S = \mathbb{Q}^{\times}$, (ii) ) $S = \mathbb{Q}^{+}$

(02) Decide which, if any , of the following are not closed with respect to the operation given, giving a specific counter-example to support your assertion.

(i) $\mathbb{Z}; a * b = \frac{a(b+1)}{2}$, (ii) $\mathbb{Q}^{+}; x * y = x^2 + y^2 - 3xy$.

(03) Let $H = \left\{\begin{pmatrix}1 & 2 & 3\\ 1 & 2 & 3\end{pmatrix}, \begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}, \begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}\right\}$ be a set. Show that $H$ is closed under Composition.

(04) Consider the binary operation $*$ defined on $\mathbb{Z}$ by $a * b = a + 2b$.
Is the operation $*$ commutative on $\mathbb{Z}$? Justify your answer.

(05) Consider the binary operation $*$ defined on $\mathbb{Z}$ by $a * b = a + 2b$.
Is the operation $*$ associative on $\mathbb{Z}$.

(06) Let $*$ be the operation defined on $\mathbb{Z}$ by $a * b = ab^2$.
Find the identity element if there exists.

(07) Construct the cayley table for $(\mathbb{Z}_5, \oplus)$. Find the inverses of each element.

(08) Consider the set $\mathbb{Z}_7^{\times} = \{1,2,3,4,5,6\}$ with respect to $\otimes$ (multiplication modulo 7).
Find the order of 2 and 3.