The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination - 2023/2024
Pure Mathematics - Level 03
PEU3301 – Foundations of Mathematics

Duration: - Two hours

Date: - 17-03-2024 **Time: - 01.30 p.m. – 3.30 p.m.**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This paper consists of **TWO** sections, Section **A** and Section **B**. Section **A** is compulsory and it consists of **SIX** Structured Essay Questions.
3. Section **B** consists of **FIVE** essay type questions and answers only **THREE** of them. Each question in Section **B** carries 100 marks.

SECTION A

Answer **ALL** the Questions.

1.

a) Let $A = \{1, 2, \{1\}, \{1, 2\}\}$. Find the power set of A.

b) Let A and B be two sets. Show that $A \setminus (A \setminus B) = A \cap B$.

c) Find the domain of the function $f(x) = \frac{x^2-4}{x-2}, x \in \mathbb{R} \setminus \{2\}$. Express your answer in interval notation.

d) Let $f(x) = 3x - 1,\ x \in \mathbb{R}$ and $g(x) = x^2 + 2,\ x \in \mathbb{R}$. Find $f \circ g$.

e) Let $f(x) = x^3,\ x \in \mathbb{R}$. Find $f([3,5])$.

f) Let $m \in \mathbb{N}$. Using mathematical induction prove that $m + n \in \mathbb{N}$ for every $n \in \mathbb{N}$.

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SECTION B

Answer **THREE** Questions **ONLY**.

2.

a) Solve the inequality $\frac{2x-5}{x-4} \geq 3$, where $x \in \mathbb{R}$.

b) Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 1$. Find $f^{-1}([2, 10))$.

c) Let A, B and C be sets. Prove that $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

3.

a) Let m be an integer. Let R be the relation defined on $\mathbb{Z}$ by xRy if m divides $x - y$.

i. Show that R is an equivalence relation on $\mathbb{Z}$.

ii. Taking $m = 2$, show that $R_0 = \{2n : n \in \mathbb{Z}\}$ and $R_1 = \{2n + 1 : n \in \mathbb{Z}\}$ are the only two equivalence classes of R.

b) Let $x, y \in \mathbb{Z}$. Prove that,

i. if x is even and y is even, then $x + y$ is even.

ii. if x is even and y is even, then xy is even.

4.

a) Let $f(x) = 4x + 5, x \in [1, 2]$. Show that f is a bijection from $[1, 2]$ into $[9, 13]$.

b) Show that the set of rational numbers $\mathbb{Q}$ is countable.

c) Let α be a transcendental number. Show that $\frac{1}{\alpha}$ is a transcendental number.

5.

a) Show that the set $\mathbb{R}$ of all real numbers is not bounded above.

b) Let $(0, 1)$ be a set.

i. Show that $(0, 1)$ is bounded.

ii. Prove that the set $(0, 1)$ does not have a maximum.

iii. Show that $\inf(0, 1) = 0$.

c) Let x be a rational number and let y be an irrational number. Show that $x - y$ is an irrational number.

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6.

a) Let $a_n = \frac{1}{2^{n-1}}$ for $n \in \mathbb{N}$.

   i. Find the n^{th} partial sum $S_n = \sum_{k=1}^{n} a_k$, for each $n \in \mathbb{N}$.

   ii. Show that the series $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} a_n = 2$.

b) Let $b_n = (-1)^{n+1}$ for $n \in \mathbb{N}$. Show that the series $\sum_{n=1}^{\infty} b_n$ diverges.

c) By using the definition for limit of a sequence prove that $\lim_{n \to +\infty} \frac{n+3}{2n+1} = \frac{1}{2}$.

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