



The Open University of Sri Lanka
Department of Mathematics
Advanced Certificate in Science Programme – Level 2
MYF2519 - Combined Mathematics I
Final Examination 2025/2026

Date: 30-08-2025

From 9:30 am. To 12:30 pm.

Answer **All** Questions in **Part A** and Answer **Five** Questions in **Part B**.

PART A

1. Find the domain of the function $f(x) = \frac{\sqrt{x-1}+1}{\sqrt{(4-x^2)}}$
2. The functions $f(x)$ and $g(x)$ are defined by $f(x) = 9x - 4$; $x \in \mathbb{R}$, and $g(x) = x^3$; $x \in \mathbb{R}$. Find the following:
 - (a) Find $f \circ g$,
 - (b) Find f^{-1} ,
 - (c) Find $f^{-1} \circ g$,
3. Solve the equation, $5^{(2x+1)} - 5^{(x+2)} - 5^x + 5 = 0$.
4. Solve the equation, $\log_3 x - 2 \log_x 3 = 1$
5. Given that the polynomial function $f(x) = 2x^4 + x^3 - x^2 + px + q$, where p, q are constants. The remainders when $f(x)$ divided by $(x - 1)$ and $(x + 1)$ are 5 and 1, respectively. Find the values of p and q .
6. Given that p is a real number. Show that the roots of the equation $x^2 + 2(p - 3)x + 2p - 12 = 0$ are real.
7. Solve the inequality $\frac{5}{5x-1} > \frac{2}{2x+1}$.
8. Find the equation of the line which passes through the point of intersection of the two straight lines $x - 2y + 3 = 0$ and $4x - y + 2 = 0$, and is parallel to the straight line $x - y + 1 = 0$.
9. Show that, $\tan^{-1} \frac{120}{119} = 2 \sin^{-1} \frac{5}{13}$.
10. Show that

$$\tan \theta + \cot 2\theta = \operatorname{cosec} 2\theta.$$

PART B

11. (a) Let $f(x) = x^4 + ax^3 + 4x + b$, where $a, b \in \mathbb{R}$. If $(x^2 + 1)$ is a factor of $f(x)$.

(i) Find the values of a and b . Find the other factor.

(ii) **Deduce** the remainder and quotient when $f(x)$ is divided by $(x^2 + 1)(x + 4)$.

(b) Given that

$$9x = A(2x + 1)(1 - x) - B(1 - x) + C(2x + 1)^2, \text{ for all } x \in \mathbb{R}.$$

Find the values of A, B and C .

Deduce the partial fractions of $\frac{9x}{(2x+1)^2(1-x)}$.

12. Let $p, q \in \mathbb{R}, q \neq 0$. Let $f(x) = x^2 - px + q$. Show that $x = 0$ is not a root of the equation $f(x) = 0$. Let α and β be the roots of $f(x) = 0$. Write down $(\alpha + \beta)$ and $\alpha\beta$ in terms of p and q .

(a) Show that the quadratic equation whose roots are $\alpha^2 + \frac{1}{\beta^2}$ and $\beta^2 + \frac{1}{\alpha^2}$ is

$$g(x) = q^2x^2 - (p^2 - 2q)(q^2 + 1)x + (q^2 + 1)^2 = 0.$$

(b) Show that if the roots of the equation $g(x) = 0$ are real and equal, then $p = 0$ or $p^2 = 4q$.

13. Sketch the graph of $f(x) = 2|x| - |x - 2|$ and $g(x) = 3|x - 2|$ in the same diagram.

(i) Hence or otherwise, find all real values of x satisfying the inequality $|x| > 2|x - 2|$.

(ii) Find the value or values of μ , when the equation $2|x| - |x - 2| = 3|x - \mu|$ has infinite many solutions.

14. (a) Write down the sine rule and cosine rule for any triangle ABC .

With the usual notation show that

$$(i) \quad a = (b - c) \cos \frac{A}{2} \cdot \operatorname{cosec} \frac{(B - C)}{2},$$

$$(ii) \quad \cot \frac{(B - C)}{2} = \frac{(b + c)}{(b - c)} \tan \frac{A}{2} \text{ provided } B \neq C.$$

Using (i) and (ii) show that $a^2 = b^2 + c^2 - 2bc \cos A$

(b) Show that $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$ and $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$.

Deduce all solutions of,

$$\sin 2\theta + \cos 2\theta = 1$$

15. (a) Write down the maximum and minimum values of $y = 4 \cos x - 2$, where $0 \leq x \leq 2\pi$.

(b) Sketch the graph of $y = 4 \cos x - 2$, where $0 \leq x \leq 2\pi$.

(c) Find all the roots which satisfy $y = 0$ and $y = 4 \cos x - 2$, when $0 \leq x \leq \pi$.

(d) Sketch the graph of $y = |4 \cos x - 2|$, in the same diagram, where $0 \leq x \leq 2\pi$

(e) Write down the number of roots which satisfy $y = 1$ and $y = |4 \cos x - 2|$, when $0 \leq x \leq 2\pi$.

16. Show that the perpendicular distance from the point (x_0, y_0) to the line $ax + by + c = 0$ is $\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$.

Deduce the distance between two lines $ax + by + c = 0$ and $ax + by + d = 0$ is $\frac{|d - c|}{\sqrt{a^2 + b^2}}$.

(a) Find the perpendicular distance from the point $\left(\frac{4}{5}, -2\right)$ to the line $5x - 12y + 11 = 0$. Also find the distance between $5x - 12y + 11 = 0$ and $5x - 12y + 24 = 0$

(b) Find the equations of two straight lines which are two units of perpendicular distance from $\left(\frac{2}{5}, -2\right)$ and parallel to the straight-line $5x - 12y - 9 = 0$.

17. Consider the two straight lines, $y + \sqrt{5}x - \sqrt{5} = 0$ and $y - \sqrt{5}x + \sqrt{5} = 0$. Show that the point $P \equiv (1, 0)$ lies on the two lines.

i) Find the bisector of the acute angle between the two straight lines.

ii) Show that the point $Q \equiv (1, 1)$ lies on the bisector of the acute angle.

iii) Find the equation of the line which is perpendicular to the bisector of the acute angle through the point $A \equiv (-1, 0)$.

iv) Find the area of the triangle PAQ.

