

The Open University of Sri Lanka
 Department of Mathematics
 Advanced Certificate in Science Programme - Level 2
 MYF2521 - Combined Mathematics 3
 Final Examination 2025/2026



Date: 14-09-2025

From: 09:30 am. To 12:30 pm.

Answer All Questions in Part A and Answer Five Questions in Part B.

PART A

- Using the Principle of Mathematical Induction, prove that $(10^n - 1)$ is divisible by 9 for all positive integers n .
- In the geometric progression, 4th, 7th and 10th terms are l, m and n . show that $m^2 = nl$.
- The coefficient of x^5 in the binomial expansion of $\left(2x + \frac{k}{2x}\right)^9$ is 128. Find the value of k where $k \in \mathbb{R}^+$
- A debating team containing 5 persons is to be chosen from a group of 7 boys and 5 girls. In how many ways can this team be formed so that it contains,
 - any 5 persons of the group.
 - at least one boy.
- Express the complex number w , $w = 1 + \sqrt{3}i$ in the modulus and argument (polar) form. Using De Moivre's theorem, find w^7 in cartesian form.
- Let $A = \begin{pmatrix} 0 & 2 & -3 \\ 0 & -1 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 3 & 4 \\ 2 & 3 \\ 1 & 2 \end{pmatrix}$.
 - Find AC .
 - Find CA .
- Evaluate the following limits.

$$(a) \lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 7}{2x^2 - 5}$$

$$(b) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \cdot \tan 9x}$$
- Parametric equation of a curve is given by $x = t(1 - t)^2$ and $y = t^2(1 - t)$, where t is a real parameter. Find $\frac{dy}{dx}$ at $t = \frac{1}{2}$.

9. Show that $\int_0^1 \frac{1}{1+e^{-x}} dx = \ln\left(\frac{1+e}{2}\right)$.

10. Find the equation of the circle that passes through the points $(1, -1)$, $(2, 0)$ and $(0, -1)$.

PART B

11. Let $U_r = \frac{1}{r(r+1)(r+2)}$. Determine the values of the real constants A , B and C such that,

$$U_r = \frac{A}{r} + \frac{B}{(r+1)} + \frac{C}{(r+2)}.$$

Show that $\sum_{r=1}^n U_r = \frac{1}{4} - \frac{1}{2(n+1)} + \frac{1}{2(n+2)}$.

Deduce that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

12. (a) The matrix A is given by $A = \begin{pmatrix} p & 1+2p \\ -1 & p-2 \end{pmatrix}$. Show that A^{-1} exists for all $p \in \mathbb{R}$.

Let $p = 3$. Write down A and A^{-1} .

Using the above results, solve the simultaneous equations

$$\begin{aligned} 3x + 7y &= 8 \text{ and} \\ -x + y &= 4 \end{aligned}$$

(b) Let $B = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$.

(i) Show that $B^2 - B - 2I = \underline{0}$, where $\underline{0}$ is 2×2 Null matrix and I is the 2×2 unit matrix.

(ii) Also Show that $B^{-1} = \frac{1}{2}(B - I)$. Deduce B^{-1} .

(iii) Show that $B^4 = 5B + 6I$.

13. (a) Let $z = x + iy$, be a complex number where $x, y \in \mathbb{R}$ and let $\bar{z}$ be the complex conjugates of z .

(i) Show that $|z|^2 = z\bar{z}$, and $z - \bar{z} = 2im(z)$.

(ii) Hence show that,

$$|z - 5i|^2 = |z|^2 - 10\text{Im}(z) + 25 \text{ and}$$

$$|1 + 5iz|^2 = 25|z|^2 - 10\text{Im}(z) + 1.$$

Deduce that if $|z - 5i|^2 = |1 + 5iz|^2$, then $|z| = 1$.

- (a) Show that the complex number $\left(\frac{5-i}{2-3i}\right)$ can be expressed in the form $\lambda(1+i)$, where λ is real. State the value of λ .

Hence show that $\left(\frac{5-i}{2-3i}\right)^6$ is imaginary and determine its value.

14. (a) Differentiate the following functions with respect to x .

$$(i) y = (1 - 4x)^2(3x^2 + 1)^4$$

$$(ii) y = \sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$(iii) y = \ln|1 + 2e^x + 3e^{-x}|$$

$$(iv) y = (1 - 4x^2) \cdot \sin^{-1} 2x$$

- (b) A closed right circular cylinder is to be made such that its volume is $1024\pi \text{ cm}^3$. Find the radius of the cylinder that will make its total surface area a minimum.

15. (a) Let $f(x) = \frac{(x^2+1)}{(x^2-1)}$, $x \neq -1, 1$. Show that $f'(x)$, the derivative of $f(x)$ is given by $f'(x) = \frac{-4x}{(x-1)^2(x+1)^2}$ for $x \neq -1, 1$.

Hence find the intervals on which $f(x)$ is increasing and $f(x)$ is decreasing. Find the turning points, vertical and horizontal asymptotes of $f(x)$ and sketch the graph of $f(x)$.

- (b) Show that the volume generated when the part of the curve $y = \sqrt{1+x^2}$ in the interval $-1 \leq x \leq 1$ is rotated by 2π angles about the x -axis is $\frac{8\pi}{3}$.

16. (a) Express $\frac{4x-1}{(2x+1)(3x-2)}$ in partial fractions and **hence** evaluate $\int \frac{4x-1}{(2x+1)(3x-2)} dx$.

(b) Using integration by parts, show that $\int_0^{\frac{\pi}{4}} x \cdot \cos 2x dx = \frac{1}{8}(\pi - 2)$.

(c) Show that the area of the region enclosed by the curves $y = x^2 + 8$, $x = 3$, $x = 6$ and the x axis is 87 square units.

17. Consider the circles $S_1 = 0$ and $S_2 = 0$ where

$$S_1 \equiv x^2 + y^2 - 4x - y + 3 = 0 \quad \text{and}$$

$$S_2 \equiv x^2 + y^2 - 2x + 6y + k = 0 \quad \text{and } k \in \mathbb{R}.$$

(i) If both circles intersect orthogonally, find the value of k .

(ii) Write down the centre and the radius of each of the two circles $S_1 = 0$ and $S_2 = 0$.

(iii) Find the equation of the circle $S_3 = 0$ which passes through the origin and the points of intersection of the two circles $S_1 = 0$ and $S_2 = 0$. Write down the centre and the radius of the circle $S_3 = 0$.

(iv) Find the equation of the tangent line which passes through the point $(1,1)$ on the circle $S_1 = 0$.