THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering
Bachelor of Science Honors in Engineering

Final Examination (2023/2024)

MHZ4553: Engineering Mathematics III

Date: 27^{th} February 2025 **Time: 09:30 a.m.-12:30 p.m.**

Instructions:

- This paper consists of EIGHT (8) questions in FOUR (4) pages.
- Answer any FIVE (5) questions ONLY. All questions carry equal marks.
- Answer for each question should commence from a new page.
- Relevant Statistic Tables are provided and all symbols are in standard notation.
- State any assumptions you required and show all your workings.
- This is a closed book test and do not use red color pen.

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1. (a) Determine the nature of all the critical points of function $f(x,y) = e^{\left(-\frac{x^3}{3}+x-y^2\right)}$. *(Marks 50)*

   (b) A cooling fin for a machine is designed to have a rectangular cross-section. If the fin's base dimensions are x and y (in cm) and the heat conduction capacity $h(x,y) = xy$, use the method of Lagrange Multipliers to minimize the surface area of the fin,

   $$S(x,y) = 2xy + 20y + 20x$$

   while maintaining the heat conduction capacity $h(x,y) = 400\text{cm}^2$. *(Marks 50)*

2. (a) Find the maximum rate of change of $f(x,y) = xe^{-y}$ at the point $(-2,0)$. *(Marks 30)*

   (b) Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.

   i. Find the matrix that diagonalize matrix A. *(Marks 40)*

   ii. Use the diagonalization of matrix A to find a formula for A^k for any $k \in \mathbb{N}$. *(Marks 30)*

3. (a) Let $u(x,y) = x^2 - y^2$ for $(x,y) \in \mathbb{R}^2$.

   i. Prove that u is a harmonic function. (*Marks 30*)

   ii. Find an analytic function f such that $\mathrm{Re}(f) = u$ and $f(0) = 0$. (*Marks 30*)

   iii. Use part (ii) to prove that $f(z) = z^2$. (*Marks 10*)

   (b) Compute $\oint_C \dfrac{3z+1}{(z-1)^3(z+4)}dz$, where $C : |z| = 3$. (*Marks 30*)

4. (a) Consider the following equation.

$$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0. \tag{1}$$

   Find the series solution y of equation (1) around $x = 0$ with $y(0) = 1$.

   (*Marks 50*)

   (b) The temperature $T(t)$ of an object changes over time according to Newton's law of cooling is given as

$$\frac{dT}{dt} = -k\left(T(t) - T_{env}(t)\right),$$

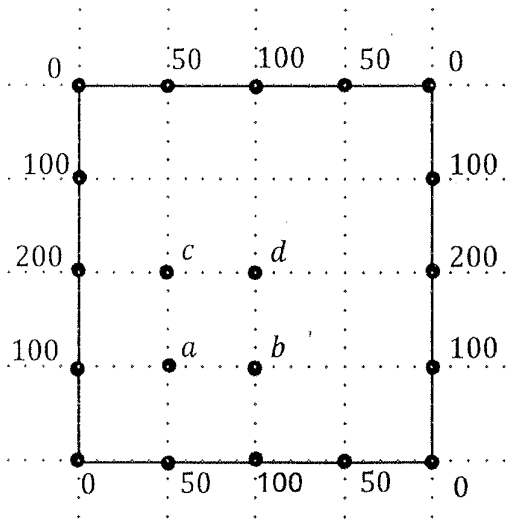
   where k is the cooling constant of the object and T_{env} is the temperature of the surrounding environment which varies with time.
   If $k = 0.1min^{-1}$ and $T_{env}(t) = 20 + 5\sin(0.5t)$, use the Euler method to find approximate temperature $T(t)$ over the time interval $t \in [0,2]$ at step size 0.5 assuming that the initial temperature of the object is 100°C. (*Marks 50*)

5. (a) Use the method of characteristics to find the solution of $u_t + xu_x = 0$ subject to the initial condition $u(x,0) = x$. (*Marks 50*)

   (b) Consider a rectangular plate of dimension 1×1 with the temperature distribution $u(x,y)$ satisfying the 2-D Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{on } 0 \le x \le 1,\ 0 \le y \le 1.$$

   Assume that the plate has the boundary temperatures shown in the following figure, where the square mesh has the same lengths.



Give four equations using the explicit scheme of finite difference method which required to find an approximate value at points a, b, c and d. No need to solve for values at each points. *(Marks 50)*

Hint:

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{h},$$

$$\left(\frac{\partial u}{\partial y}\right)_{i,j} = \frac{u_{i,j+1} - u_{i,j}}{k}$$

where h and k are step sizes of variables x and y with respectively.

6. (a) Let $\vec{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \vec{u}_2 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}, \vec{v} = \begin{pmatrix} 4 \\ -3 \\ 4 \end{pmatrix}$ and $W \subset \mathbb{R}^3$ such that $W = span(\vec{u}_1, \vec{u}_2)$.

i. Find a basis for W consisting of two orthogonal vectors. *(Marks 30)*

ii. Does $\vec{v} \in W$? Justify your answer. *(Marks 30)*

(b) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a transformation defined by $T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + y \\ z + 2 \end{pmatrix}$. Is T a linear transformation? Justify your answer. *(Marks 40)*

7. (a) Define the following term used in confidence interval for proportion.

   i. The standard error of proportion. (*Marks 05*)

   ii. The margin of error. (*Marks 05*)

   (b) A statistical study is done to identify the proportion of failures in electrical components, due to receive a high voltage. Out of 1500 components 145 of components failed due to receiving high voltage.

   i. Find the point estimator of the true proportion of components which will fail due to receiving high voltage. (*Marks 15*)

   ii. Find the standard error of the proportion. (*Marks 15*)

   iii. Find the 90% confidence interval for the true proportion of components which will fail due to receive high voltage. (*Marks 25*)

   iv. State any assumptions that you made in part (iii). (*Marks 10*)

   v. Find the probability that the true proportion of components which will fail due to receiving high voltage is between 0.0817 to 0.1117. (*Marks 25*)

8. A mechanical engineer is investigating the effect of temperature (X, in $^{\circ}C$) on the viscosity (Y, in $mPa.s$) of a lubricant. The following data was collected:

Temperature (X, in $^{\circ}C$)	10	20	30	40	50	60	70	80	90	100
Viscosity (Y, in $mPa.s$)	118	115	105	100	85	80	75	65	55	50

   (a) Calculate the correlation coefficient for the above data. Interpret your answer. (*Marks 30*)

   (b) Find the coefficient of the equation of the best fitted simple linear regression model of the above data. (*Marks 30*)

   (c) Interpret the found regression coefficient in part (b). (*Marks 10*)

   (d) Predict the viscosity when the temperature is 50°C using the linear model you found in part (b). (*Marks 10*)

   (e) Calculate the residual for the observed viscosity at 50°C. (*Marks 10*)

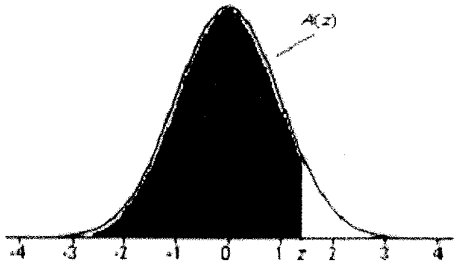
   (f) If the model is used to predict viscosity at 500°C, explain one limitation of this prediction. (*Marks 10*)

End.

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Z-table

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

t Distribution: Critical Values of *t*

		Significance level					
Degrees of freedom	*Two-tailed test:*	10%	5%	2%	1%	0.2%	0.1%
	One-tailed test:	5%	2.5%	1%	0.5%	0.1%	0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291