

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering



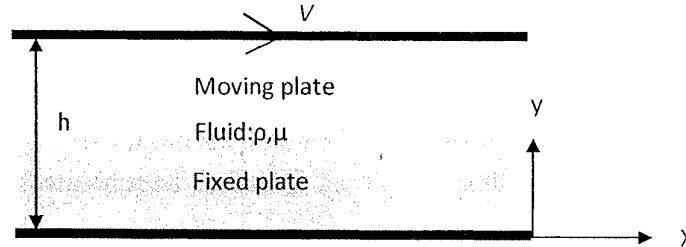
Study Programme	: Bachelor of Science Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: CVX5242 Mechanics of Fluids
Academic Year	: 2023/2024
Date	: 10 th March 2024
Time	: 0930-1230 hrs
Duration	: 03 hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **SIX (06)** questions on **Five (05)** pages.
3. Answer **Any FIVE (05)** questions.
4. Answer for each question should commence from a new page.
5. Necessary additional information is provided.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear handwriting.
8. Do not use a Red colour pen.
9. Students are expected to know the standard denominations used for equations
10. Students are allowed to use non-programmable calculators

Question (1)

Consider steady, incompressible, laminar flow of a Newtonian fluid in the narrow gap between two infinite parallel plates as shown in the Figure 01. The top plate is moving at a speed V , and the bottom plate is stationary. The distance between these two plates is h , and the gravity acts in the negative z -direction (into the paper). There is no applied pressure other than hydrostatic pressure due to gravity.

**Figure 01**

- Using continuity equation show that $\frac{\partial u}{\partial x} = 0$ for this flow field. State all the assumptions made on this flow field in deriving this equation. (4 Marks)
- Using Navier-Stokes equation, show that $\frac{d^2 u}{dy^2} = 0$ for this flow field. Please state all the assumptions made on this flow field in deriving this equation. (4 Marks)
- If the pressure at $z=0$ is P_0 , show the pressure field of the flow field is given by

$$P = P_0 - \rho g z$$

(6 Marks)

- Using boundary conditions ($u = 0$ at $y = 0$) and ($u = V$ at $y = h$) show that the velocity field is given by

$$u = V \frac{y}{h}$$

(6 Marks)

Question (2)

- Using Navier Stokes equations, derive the following boundary layer equations for a steady, laminar and two-dimensional incompressible flow moving over a flat thin plate. You may make necessary assumptions for the derivation. (10 Marks)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{dp}{dy} = 0$$

b) The velocity field $V = (u, v) = f(x, y)$ in a steady laminar boundary layer is driven by following two equations. Derive these two equations using the equations derived under part (a). U is the inviscid velocity of the horizontal flow. (10 Marks)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Question (3)

a) Using a diagram which shows the laminar boundary layer, define the displacement thickness of a boundary layer. (6 Marks)

b) Using the definition of displacement thickness provided in part (a), show that the displacement thickness is given by the following equation. (8 Marks)

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

where;

δ^* : Displacement thickness

u : Horizontal velocity at a height of y

U : Inviscid velocity

c) Define the momentum thickness (θ) for a boundary layer and derive the following equation for it. (6 Marks)

$$\theta = \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$$

Question (4)

Consider the steady, two-dimensional, incompressible velocity field defined by

$$\vec{V} = (u, v) = (ax + b)\vec{i} + (-ay + cx)\vec{j}.$$

a) Check whether the flow field satisfies the two dimensional, incompressible continuity equation. (4 Marks)

b) Using the Navier-Stokes equations with necessary assumptions, derive the x-momentum equation;

$$\frac{\partial p}{\partial x} = \rho(-a^2x - ab). \quad (5 \text{ Marks})$$

c) Using the Navier-Stokes equations with necessary assumptions, derive the y-momentum equation;

$$\frac{\partial p}{\partial y} = \rho(-a^2y - bc). \quad (5 \text{ Marks})$$

d) Using the equations derived in parts (a) and part (b), show that the pressure field for the flow is given by the following equation;

$$P_{(x,y)} = \rho \left(-\frac{a^2 x^2}{2} - \frac{a^2 y^2}{2} - abx - bcy \right) + C_1 \quad (6 \text{ Marks})$$

where C_1 is a arbitrary constant.

Question (5)

The following equation is the prandtl's boundary layer equation for laminar flow over a plate;

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} = -\frac{1}{\rho} \frac{dp}{dx} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$

a) Using the above equation and the continuity equation, show that;

$$\frac{d(U^2 \theta)}{dx} + \delta^* U \frac{dU}{dx} = \frac{\tau_0}{\rho}$$

where

θ is the momentum thickness

τ_0 is the shear stress on the plate when $y=0$

ρ is the density of fluid

U is the Inviscid velocity

(10 Marks)

b) For a laminar flow over a flat plate, a reasonable assumption for the velocity is polynomial in $\eta=y/\delta$. Assuming that the velocity distribution in the laminar boundary layer without a pressure gradient to be a fourth-degree polynomial, show that

$$\frac{u}{U} = f(\eta) = 2\eta - 2\eta^3 + \eta^4$$

(10 Marks)

Question 6

(a) An unsteady incompressible flow field is given by, $u = t^2 + 3y$ and $v = 4t + 5x$. Determine the acceleration at the point (4,3) at time $t = 3$.

(06 marks)

(b) Confirm that the following flow exists and obtain the associated stream function;

$$u = -cx/y,$$

$$v = c \ln xy$$

(08 marks)

(c) A velocity potential for a 2-dimensional flow is given by $\phi = (x^2 - y^2) + 3xy$. Compute the flow rate between the streamlines passing through points (1,1) and (2,3).

(06 marks)