

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering



080

Study Programme	: Bachelor of Science Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: CVX5443 Structural Analysis
Academic Year	: 2023/24
Date	: 19 th March 2025
Time	: 09:30-12:30hrs
Duration	: 3 hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **EIGHT (8)** questions in **Six (6)** pages.
3. Answer **ANY FIVE (05)** questions and each question carries **20 Marks**.
4. Answer for each question should commence from a **new page**.
5. This is a **Closed Book Examination**.
6. Answers should be in clear hand writing.
7. Do not use Red colour pen.
8. An electronic non-programmable calculator may be used.
9. Unless otherwise specified, standard nomenclature is used.

QUESTION 1

The stress at a point inside a continuum is given by the stress tensor (Unit of MPa),

$$\sigma_{ij} = \begin{bmatrix} 1 & -1 & \alpha \\ \beta & -5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

- (i) Determine α and β values (2 Marks)
- (ii) Determine three stress invariants after substituting α and β values (6 Marks)
- (iii) Show that principal stresses are -5.16 MPa, 1.16 MPa and 4 MPa (3 Marks)
- (iv) Draw the stress state in a Mohr's circle diagram (2 Marks)
- (v) Obtain volumetric and deviatoric stress tensors (3 Marks)
- (vi) What should be the **minimum uniaxial yield strength** of the material so that it **does not fail** with a **factor of safety of 2.0**, according to the **Von Mises** criteria (4 Marks)

QUESTION 2

- (i) Briefly explain why **Airy's stress function** is useful in structural analysis (3 Marks)
- (ii) A cantilever beam with rectangular cross section is shown below (Figure Q2). It is loaded with two concentrated loads: P_1 and P_2 at the free end of the beam. The following mathematical function has been proposed as the Airy's stress function.

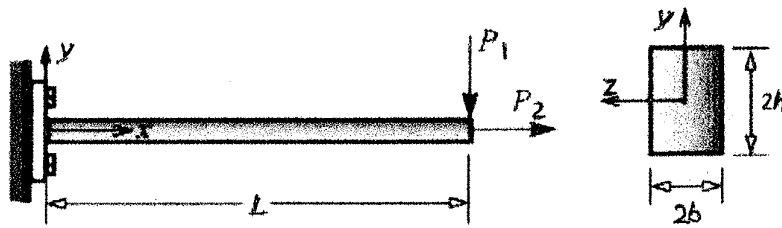


Figure Q2

$$\phi = a_1 \left[3 \left(\frac{y}{h} \right) - \left(\frac{y}{h} \right)^3 \right] (x - L) + a_2 \left(\frac{y}{h} \right)^2$$

Where a_1 and a_2 are constants.

- (a) Show that above function satisfies the bi-harmonic equation (3 Marks)
- (b) Determine stress components in terms of b, h, L, P_1, P_2, x and y (4 Marks)
- (c) Determine components a_1 and a_2 stating the relevant boundary conditions (5 Marks)
- (d) Calculate two displacement components (u_x and u_y) (5 Marks)

QUESTION 3

- (i) Describe Mode I, Mode II and Mode III cracks in fracture mechanics using clear sketches (4 Marks)
- (ii) A single degree of freedom system has a mass of 40 kg and spring constant of 350 N/m. System is given an initial displacement of 10mm and initial velocity is 100mm/s. Find
- (a) The natural frequency (4 Marks)
 - (b) The period of vibration (4 Marks)
 - (c) The amplitude of vibration (4 Marks)
 - (d) The time at which third maximum will occur (4 Marks)

QUESTION 4

- (i) Briefly explain how **Surface Coating technique** is used in the crack investigation. (2 Marks)
- (ii) Briefly explain the difference between **metal foil type** and **bonded wire type electrical resistance strain gauges**. (3 marks)
- (iii) The stress state of a certain concrete component was determined using a strain rosette as shown in Figure Q4. Due to the loadings, strain gauges gave strain values as $\epsilon_a = -50 \mu\epsilon$, $\epsilon_b = 60 \mu\epsilon$, $\epsilon_c = 90 \mu\epsilon$.

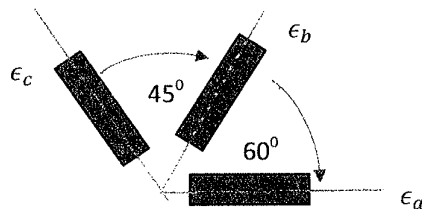


Figure Q4

- (a) Draw the Mohr's circle for strains (4 marks)
- (b) Determine the in-plane principal strains. (5 marks)
- (c) Determine the principal stresses, if the elastic modulus and Poisson's ratio of concrete material are 25 GPa and 0.20, respectively. (6 marks)

QUESTION 5

- (i) Explain three characteristics of statically indeterminate structures (4 Marks)
- (ii) A continuous beam (ABCD) is shown in Figure Q5. Flexural rigidities of members AB and CD are equal to $2EI$ and member BC is $4EI$. Uniformly distributed load ($2W$) is acting on member, AB. There is a concentrated load ($2WL$) in member CD.
- (a) Determine the degree of statical indeterminacy of the beam. (3 Marks)
- (b) Draw a released structure. (3 Marks)
- (c) Determine the **flexibility matrix** for the drawn released structure. (4 Marks)
- (d) Explain how do you solve the problem and find bending distribution (6 Marks)

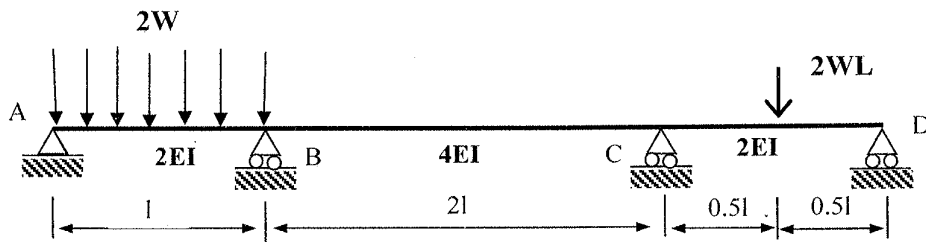


Figure Q5

QUESTION 6

Analyse the continuous beam shown in Figure Q6 using displacement method of structural analysis using following steps

- (i) Find kinematic indeterminacy of the structure (3 Marks)
- (ii) Draw the structure with independent nodal displacements (4 Marks)
- (iii) Determine the **stiffness matrix** of the structure. (7 Marks)
- (iv) Obtain the bending moment distribution of the beam (6 Marks)

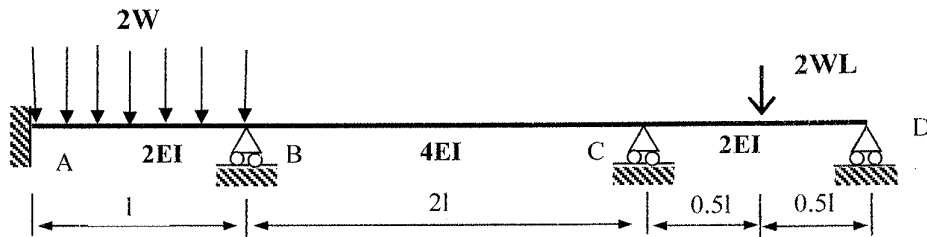


Figure Q6

QUESTION 7

- (i) List two assumptions used in theory of thin plates under small deflections (3 Marks)
- (ii) A Circular plate of radius "a" carries a uniformly distributed load "q". The edge at "r = a", is fixed supported as shown in Figure Q7.
- (a) Show that the deflection of the plate is given by $w = \frac{q}{64D} (a^2 - r^2)^2$ (12 Marks)
- (b) Find the bending moment at the fixed support (5 Marks)

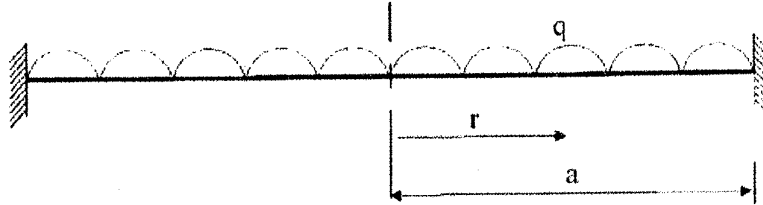


Figure Q7

Hint: Uniformly loaded solid circular plate, with symmetrical boundary conditions is given as

$$\nabla^4 w = \frac{1}{r} \frac{d}{dr} \left\{ r \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) \right] \right\} = \frac{q_0}{D}$$

Where q_0 is a constant.

QUESTION 8

- (i) Determine the shape factor for a rectangular cross section (3 Marks)
- (ii) A two-storey frame structure is shown in Figure Q8. Dimensions and plastic moments of the columns and beam are given in the figure. Plastic moment (M_p) can be assumed to $3wl^2/4$.
- (a) Draw possible locations of plastic hinge formations. (3 Marks)
- (b) Determine number of elementary mechanisms (4 Marks)
- (c) Draw elementary failure mechanisms. (4 Marks)
- (d) Determine load factors for each elementary failure mechanism. (6 Marks)

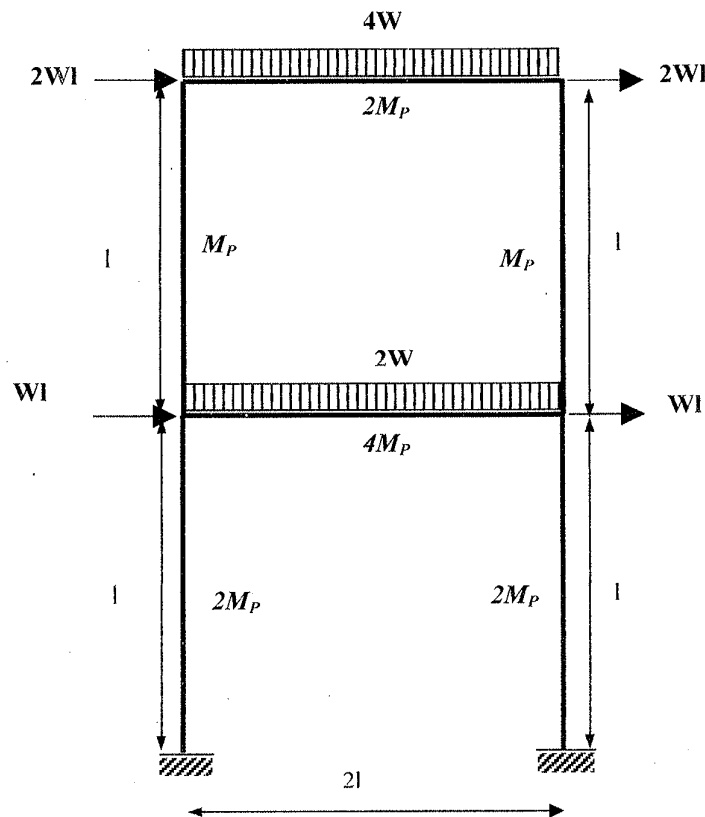
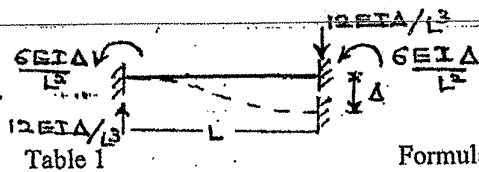


Figure Q8 (Not to scale)

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Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Cantilever Beam				
	0	M_0	$\theta_A = \frac{M_0 L}{EI}$	$Y_A = \frac{M_0 L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_0}{2L}$	$M_B = -\frac{M_0}{2}$	$\theta_A = -\frac{M_0 L}{4EI}$	$Y_{\max} = \frac{M_0 L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{3M_0}{2L}$	$M_B = -\frac{3WL}{16}$ $M_C = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{\max} = 0.00962 \frac{WL^3}{EI}$ at $x = 0.447L$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Y_0 = \frac{Wa^2 b^2}{12EIL^3}(3L)$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{\max} = 0.0054 \frac{WL^3}{EI}$ at $x = 0.42L$
	$S_A = +\frac{WL}{10}$	$M_{\max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{\max} = 0.00239 \frac{WL^3}{EI}$ at $x = 0.44L$
	$S_A = +\frac{11WL}{40}$	$M_{\max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{\max} = 0.00305 \frac{WL^3}{EI}$ at $x = 0.40L$