00079

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical and Computer Engineering

Study Programme	: Bachelor of Science Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: EEX5360 Signals and Systems**
Academic Year	: 2023/24
Date	: 12th March 2025
Time	: 1330 – 1630 hrs
Duration	: **03 hours**

General Instructions

1. Read all instructions carefully before answering the questions.

2. This question paper consists of **five (05)** *compulsory questions* in **three (03)** pages and the **formula sheet** is provided in **page 3-5**.

3. Calculators are permitted to use.

4. This is a Closed Book Test (CBT).

5. Do not use a red colour pen to write the answers.

Question 1 **[20 marks]**

1. Figure Q1 shows a passive RLC circuit with a current source.

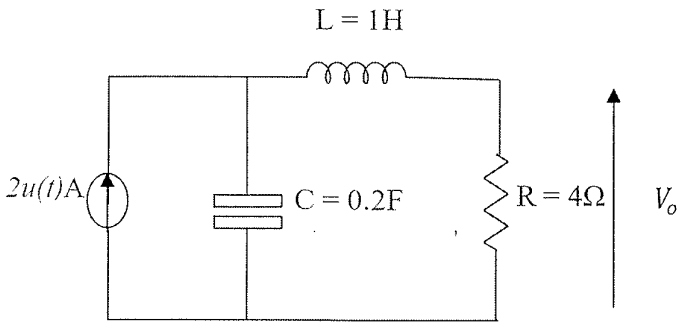


Figure Q1: RLC circuit.

i. Transform the circuit elements from the time domain to the Laplace domain. [06 marks]

ii. Hence, draw the Laplace domain equivalent circuit for Figure Q1. [04 marks]

iii. Find the output voltage (V_0) across the resistor. [10 marks]

Question 2 **[20 marks]**

a) A periodic triangular signal is shown in Figure Q2. Find its complex Fourier series. [08 marks]

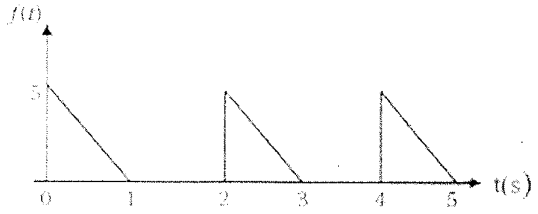


Figure Q2: Periodic Triangular wave.

b) Explain the significance of the impulse response in a discrete-time linear time-invariant (LTI) system. [02 marks]

c) Consider a discrete-time LTI system where the input signal is $x[n] = 0.3^n\, u[n]$, and the impulse response is $h[n] = (-0.4)^n\, u[n]$, where $u[n]$ is the unit step function. Find the output $y[n]$ of the system. [10 marks]

Hint*: You may use the Geometric series identity $\sum_{K=0}^{n}\left(\frac{a}{b}\right)^k = \frac{1-\left(\frac{a}{b}\right)^{n+1}}{1-\left(\frac{a}{b}\right)}$

Question 3 **[20 marks]**

The *LCR* circuit of Figure Q3 consists of a resistor R, a capacitor C and an inductor L connected in series together with a voltage source $e(t)$. Before closing the switch at time $t = 0$, both the charge on the capacitor and the resulting current in the circuit are zero. Given that $R = 160\ \Omega$, $L = 1\text{H}$, $C = 10^{-4}$ F and $e(t) = 20$ V.

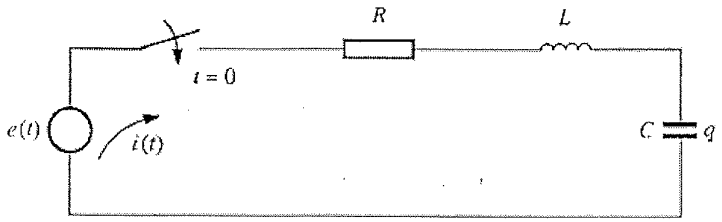


Figure Q3: RLC circuit.

i. Applying Kirchoff's second law to the above circuit, obtain the characteristic equation on the time domain for the charge *q(t)*. [05 marks]

ii. Applying the initial conditions, obtain the transfer function $Q(s)$ in Laplace domain.

iii. Determine the charge q(t) across the capacitor at time *t=0*. State any assumptions you make. [05 marks]

Hint: Take the inverse Laplace Transformation of $Q(s)$ *in ii.* [07 marks]

iv. Hence Find the current $i(t)$ across the capacitor. [03 marks]

Hint: Use $i(t) = \frac{dq(t)}{dt}$

Question 4 **[20 marks]**

(a) Using the Z Transformation, solve the following second order differential equation.

$$y[k+2] - 5y[k+1] + 6y[k] = 0$$

Subjected to the initial conditions, $y(0) = 1$ and $y(1) = 2$.

[10 marks]

(b) Using the Z Transformation, solve the following first order differential equation.

$$2f[k] - 3f[k+1] = (-1)^{k-1}$$

Subjected to the initial conditions, $f(0) = 2$. [10 marks]

Question 5 **[20 marks]**

A block diagram of a digital signal processing (DSP) system is given by Figure Q5 where a continuous-time signal $x(t)$ is sampled by a Continuous-to-Discrete (C/D) converter, producing a discrete-time signal $x[n]$. The discrete-time system $H[n]$ processes $x[n]$, generating $y[n]$. The Discrete-to-Continuous (D/C) converter then reconstructs the continuous-time output $y(t)$. The sampling period T is the time interval between consecutive samples, with $f_s = \frac{1}{T}$ as the sampling frequency.

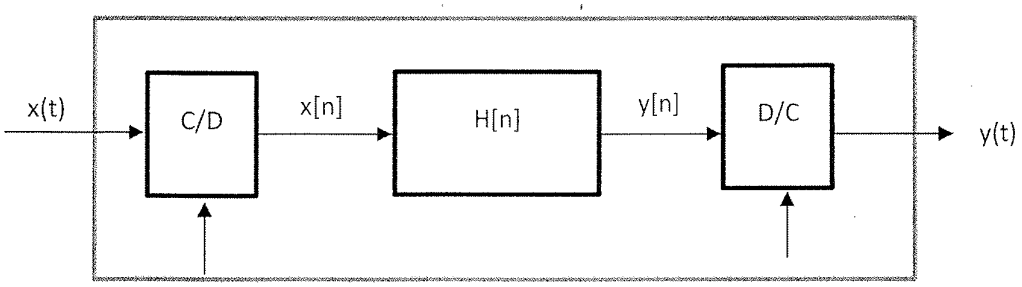


Figure Q5: Block diagram.

Considering the above block diagram, where T = 1/250 seconds and the input to this diagram is

$$x(t) = 4\cos(300\pi t) + 2\sin(600\pi t)$$

And h[n] is an ideal low pass filter with cutoff frequency $\omega_c = \frac{8\pi}{5}\frac{rad}{sec}$.

a) State and plot the continuous time Fourier transform of x(t), denoted as X(jω). [05 marks]

b) State and plot the discrete time Fourier transform of x[n], denoted as $X(e^{j\omega})$. [05 marks]

c) State and plot the continuous time Fourier transform of y[n], denoted as $X(e^{j\omega})$. [05 marks]

d) State and plot the continuous time Fourier transform of y(t), denoted as Y(jω). [05 marks]

Formula sheet

1. Exponential Fourier series expansion and its components

Exponential Fourier Series

$$x(t) = \sum_k C_k e^{jk\omega_0 t}$$

where $C_k = \frac{1}{T_0}\int_0^{T_0} x(t).e^{-jk\omega_0 t}dt$ and $C_0 = \frac{1}{T_0}\int_0^{T_0} x(t)dt$

Deviation property of Fourier coefficient

$$X(t) \quad \Leftrightarrow \quad Ck$$

$$\frac{dX(t)}{dt} \Leftrightarrow jk\omega_0 \, Ck$$

Sum of Series

$$S_n = \sum_{n=0}^{N} ar^n = a + ar + ar^2 + \cdots + ar^N = \frac{a(1 - r^{N+1})}{1 - r}$$

2. Laplace Transformation-Important Formulas

$$L\{\delta(t)\} = 1$$

$$L\{\delta(t - a)\} = e^{-as}$$

2.1 Ordinary Differential Equation Formulas

$$L\left\{\frac{d^2x}{dt^2}\right\} = s^2X(s) - sx(0) - \dot{x}(0)$$

$$L\left\{\frac{dx}{dt}\right\} = sX(s) - x(0)$$

2.2 Second Shift Theorem

$$L[f(t - a) + u(t - a)] = e^{-as}F(s)$$

2.3 Inverse of the Second Shift Theorem

$$L^{-1}[fe^{-as}F(s))] = f(t - a) + u(t - a)$$

2.4 Laplace Transform Pairs

f(t)	F(s)
$u(t)$	$\frac{1}{s}$
$r(t) = tu(t)$	$\frac{1}{s^2}$
$e^{-at}u(t)$	$\frac{1}{s + a}$
$t^n e^{-at}u(t)$	$\frac{n!}{(s + a)^{n+1}}$
$sin\omega_0 t \, u(t)$	$\frac{1}{s^2 + {\omega_0}^2}$
$cos\omega_0 t \, u(t)$	$\frac{s}{s^2 + {\omega_0}^2}$

3. Z Transformation-Important Formulas

Z Transform Pairs and Properties

Z Transform Pairs		
Time Domain *	Z Domain	
	z	z^{-1}
δ[k] (unit impulse)	1	1
γ[k] * (unit step)	$\Gamma(z)=\frac{z}{z-1}$	$\Gamma(z)=\frac{1}{1-z^{-1}}$
a^k	$\frac{z}{z-a}$	$\frac{1}{1-z^{-1}a}$
e^{-bTk}	$\frac{z}{z-e^{-bT}}$	$\frac{1}{1-z^{-1}e^{-bT}}$
k	$\frac{z}{(z-1)^2}$	$\frac{z^{-1}}{(1-z^{-1})^2}$
sin(bk)	$\frac{z\sin(b)}{z^2-2z\cos(b)+1}$	$\frac{z^{-1}\sin(b)}{1-2z^{-1}\cos(b)+z^{-2}}$
cos(bk)	$\frac{z(z-\cos(b))}{z^2-2z\cos(b)+1}$	$\frac{1-z^{-1}\cos(b)}{1-2z^{-1}\cos(b)+z^{-2}}$
a^ksin(bk)	$\frac{az\sin(b)}{z^2-2az\cos(b)+a^2}$	$\frac{az^{-1}\sin(b)}{1-2az^{-1}\cos(b)+a^2z^{-2}}$
a^kcos(bk)	$\frac{z(z-a\cos(b))}{z^2-2az\cos(b)+a^2}$	$\frac{1-az^{-1}\cos(b)}{1-2az^{-1}\cos(b)+a^2z^{-2}}$

Z Transform Properties	
Property Name	Illustration
Linearity	$af_1[k]+bf_2[k]\xleftrightarrow{Z}aF_1(z)+bF_2(z)$
Left Shift by 1	$f[k+1]\xleftrightarrow{Z}zF(z)-zf[0]$
Left Shift by 2	$f[k+2]\xleftrightarrow{Z}z^2F(z)-z^2f[0]-zf[1]$
Left Shift by n	$f[k+n]\xleftrightarrow{Z}z^nF(z)-z^n\sum_{k=0}^{n-1}f[k]z^{-k}$ $=z^n\left(F(z)-\sum_{k=0}^{n-1}f[k]z^{-k}\right)$
Right Shift by n	$f[k-n]\xleftrightarrow{Z}z^{-n}F(z)$
Multiplication by time	$kf[k]\xleftrightarrow{Z}-z\frac{dF(z)}{dz}$
Scale in z	$a^kf[k]\xleftrightarrow{Z}F\left(\frac{z}{a}\right)$
Scale in time	$f\left[\frac{k}{n}\right]\xleftrightarrow{Z}F(z^n)$; n is an integer, $n\geq 1$
Convolution	$f_1[k]*f_2[k]\xleftrightarrow{Z}F_1(z)F_2(z)$
Initial Value Theorem	$f[0]=\lim_{z\to\infty}F(z)$
Final Value Theorem (if final value exists)	$\lim_{k\to\infty}f[k]=\lim_{z\to 1}(z-1)F(z)$

4. Fourier Transform -Important Formulas

Fourier Transform Pairs

X(t)	X(jω)
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$e^{-at}u(t)$	$\frac{1}{j\omega+a}$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(j\omega+a)^n}$
$sin\omega_0 t$	$j\pi\delta(\omega+\omega_0)-j\pi\delta(\omega-\omega_0)$
$cos\omega_0 t$	$\pi\delta(\omega+\omega_0)-j-\pi\delta(\omega-\omega_0)$
$x(t-t_0)$	$X(j\omega)e^{-j\omega t_0}$

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