THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering
Bachelor of Science Honors in Engineering

Final Examination (2023/2024)
MPZ5554: Engineering Mathematics IV

Date: 09th March 2025 (Sunday) **Time: 13:30 –16:30**

Instructions:

- Part A is compulsory.
- Part B contains 07 (seven) questions. Answer **only 05 (five)** from part B.
- All the symbols are in standard notation unless they are defined.

PART A

1. Evaluate $\iint dA$ over the region in the first quadrant for which $-x + y \leq 2, x = 1$. *(10 marks)*
2. Let $\boldsymbol{F} = y\boldsymbol{i} + z\boldsymbol{j} + x^2\boldsymbol{k}$. Find $\nabla \cdot \boldsymbol{F}$ and hence evaluate $\iint (\boldsymbol{F} \cdot \boldsymbol{n}) dA$. *(10 marks)*
3. Let $\boldsymbol{F} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$. Find $\nabla \times \boldsymbol{F}$ and hence evaluate $\int \boldsymbol{F} \cdot d\boldsymbol{r}$. *(10 marks)*
4. Let $z \in \mathbb{C}$. Determine the nature of singularity of the function $f(z) = \frac{e^z - 1}{z}$. *(10 marks)*
5. Write down the Maclaurin series expansion of the function $f(z) = \frac{1}{2+4z}, z \neq -\frac{1}{2}$. *(10 marks)*
6. Sketch the image of the line $y = 1$ under the mapping $w = z^2$. *(10 marks)*
7. Find the Laurent series expansion of $f(z) = \frac{1+z}{z}$ at $z = 0$. *(10 marks)*
8. Find the points of feasible solutions of the Linear Programming problem which having the following constraints, Subject to :

$$3x + 7y \leq 24$$
$$3x + 6y \leq 21$$
$$x \geq 1$$
$$y \geq 2$$
$$x, y \geq 0$$

(10 marks)

9. In a Maximization Linear Programming problem if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then comment about the optimal solution of this problem. *(10 marks)*

10. Let it is given that null hypothesis associate with and ANOVA was rejected at 1% significant level, then what is the decision at 5% significant level? *(10 marks)*

PART B

Q1

a. Consider the following map,

$$u = 2x + 3y, \qquad v = 4y.$$

i. Consider the unit square with vertices of (0,0), (1,0), (0,1), (1,1) in the xy-plane. Find the image of the vertices of the unit square. *(10 marks)*

ii. Find the area of the image of the unit square under the above transformation.

[Hint: Take $dxdy = \left|det\left((DT^{-1})(u,v)\right)\right|dudv$ and $(DT^{-1})(u,v) = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix}$]

(30 marks)

b. Verify Gauss divergence theorem for

$$\boldsymbol{F} = y\boldsymbol{i} + x\boldsymbol{j} + z^2\boldsymbol{k}$$

taken over the region bounded by $x^2 + y^2 \leq 1$, and $0 \leq z \leq 2$.

(60 marks)

Q2

a. Verify Stoke's theorem for the vector field,

$$\boldsymbol{F} = x^2\boldsymbol{i} + 2x\boldsymbol{j} + z^2\boldsymbol{k}$$

on the ellipse which is bound by the surface $S = \{(x,y,z): 4x^2 + y^2 \leq 4,\ z = 0\}$. Take the circulation in the counterclockwise sense.

(60 marks)

b. Verify Green's theorem for the disk with center (0,0) and radius 1 for the functions $P(x,y) = x + y$, $Q(x,y) = y$ with the Green's theorem $\iint_A \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \oint_C Pdx + Qdy$.

(40 marks)

Q3

a. Find the Laurent series expansions of the function,

$$f(z) = \frac{1}{(z-1)(z-3)} \; ; z \in \mathbb{C}\backslash\{1,3\}$$

at $z_0 = 0$ in each of the following domains.

i. $0 < |z| < 1$ *(10 marks)*

ii. $1 < |z| < 3$ *(10 marks)*

iii. $|z| > 3$ *(10 marks)*

b. Write down the root test for convergence. *(05 marks)*

i. Find a strip $\{z \in \mathbb{C}: -k < \mathrm{Im}(z) < k\}$ in which the series $\sum_{n=1}^{+\infty} \frac{1}{3^n} \sin nz$ is convergent. *(30 marks)*

ii. What is the largest possible value of k? *(10 marks)*

iii. Prove that the series defines an analytic function of z in the strip $\{z \in \mathbb{C}: -k < \mathrm{Im}(z) < k\}$. *(25 marks)*

[Hint: Use $|e^{iz}| = e^{-\mathrm{Im}(z)}$].

Q4

a.

i. Define the direction cosine of a vector with respect to a three-dimensional vector $\boldsymbol{a}$. *(10 marks)*

ii. Let $OXYZ$ is a 3 – Dimensional cartesian coordinate system and $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are the unit vectors along the axes OX, OY and OZ respectively. Suppose we rotate the cartesian frame by an angle θ about the origine of it in the sense of anticlockwise direction. Find the new positions of the axes OX, OY and OZ with respect to the first position of $OXYZ$ (before rotation) in terms of θ. *(40 marks)*

b. The moment of inertia tensor $I_{i,j}$ is given by,

$$I_{i,j} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 7 \end{bmatrix}; \quad a, b, c \in \mathbb{R}$$

find

i. Eigen values and eigenvector matrix of $I_{i,j}$. *(30 marks)*

ii. Matrix of tensor components referred to the principal axes. *(10 marks)*

iii. Verify the result in the above part (ii), by using the Transformation Law of tensors. *(10 marks)*

Q5

a. Use the method of separation of variables to solve the partial differential equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{4}\frac{\partial u}{\partial t}$$

Given that $u(t, 0) = u(t, 1) = 0$ and $u = \sin \pi x + \sin 2\pi x$ when $t \to 0$. *(50 marks)*

b. Using the cosine Fourier Transform, solve the partial differential equation,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 \leq x < \infty, \qquad t > 0$$

Subject to the conditions,

i. $u(x, 0) = 0$ for $x \geq 0$.

ii. $\frac{\partial u}{\partial x}(0, t) = -1$

(50 marks)

Q6

An electrical engineer is tasked with designing a power distribution system for a small industrial complex. The system must supply a certain amount of power demand while minimizing cost and ensuring operational constraints are met. The engineer has two types of power sources available, where source A supplies 3 MW per unit and Costs $4,000 per unit, and source B supplies 5 MW per unit and Costs $6,500 per unit. The total power demand must be at least 1200 MW but not exceed 1800 MW to maintain efficiency and avoid overload. The goal is to minimize cost while meeting power supply constraints.

a. Identify the decision variables. *(10 marks)*

b. Formulate a mathematical model (Linear Programming Model) to estimate the production structure which will minimize the cost. *(25 marks)*

c. Convert the linear programming problem in part (b) to corresponding standard form of the LP problem. *(15 marks)*

d. Use M method to find the optimal solution.

   i. Modify standard form of the LP problem into a form which can be used for M method. *(15 marks)*

   ii. Use simplex method to find the optimal power distribution plan to minimize the cost. *(35 marks)*

Q7

A civil engineering firm is designing a new pedestrian bridge and needs to select the most durable construction material. The firm is considering five different materials and wants to determine which one provides the highest load-bearing capacity before failure. To make an informed decision, engineers conduct a series of tests on sample beams made from each material. To prevent unexpected failures, the firm tests samples of five materials under controlled conditions to measure the maximum load (in kilonewtons) each sample can withstand before breaking. The engineers record the maximum load-bearing capacity (in kN) of different samples for each material:

Brand A	Brand B	Brand C	Brand D	Brand E
320	280	260	220	220
350	310	290	250	290
330	295	270	270	220
300	275	265	260	290
310	-	255	-	300

If the engineers claim that all the brands could load the same bearing capacity,

a. Identify the factor variable and dependent variable. *(10 marks)*

b. State the null and alternative hypothesis associated with the engineers' claim. *(50 marks)*

c. Test the hypothesis about the engineers claim at a significant 5% level using ANOVA table and interpret the findings. *(20 marks)*

d. What can you say about the engineers' claim at a significant 10% level. *(10 marks)*

e. State the assumptions you made for this test. *(10 marks)*

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F Distribution: Critical Values of *F* (5% significance level)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.89	1.85	1.81	1.78
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.86	1.82	1.78	1.75
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.84	1.79	1.75	1.72
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.82	1.77	1.73	1.70
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.80	1.76	1.72	1.69
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.79	1.75	1.71	1.68
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.78	1.73	1.69	1.66
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.76	1.71	1.67	1.64
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.74	1.69	1.66	1.62
250	3.88	3.03	2.64	2.41	2.25	2.13	2.05	1.98	1.92	1.87	1.79	1.73	1.68	1.65	1.61
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.78	1.72	1.68	1.64	1.61
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85	1.78	1.72	1.67	1.63	1.60
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.77	1.71	1.66	1.62	1.59
600	3.86	3.01	2.62	2.39	2.23	2.11	2.02	1.95	1.90	1.85	1.77	1.71	1.66	1.62	1.59
750	3.85	3.01	2.62	2.38	2.23	2.11	2.02	1.95	1.89	1.84	1.77	1.70	1.66	1.62	1.58
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.76	1.70	1.65	1.61	1.58

F Distribution: Critical Values of *F* (5% significance level)

ν_2 \ ν_1	25	30	35	40	50	60	75	100	150	200
1	249.26	250.10	250.69	251.14	251.77	252.20	252.62	253.04	253.46	253.68
2	19.46	19.46	19.47	19.47	19.48	19.48	19.48	19.49	19.49	19.49
3	8.63	8.62	8.60	8.59	8.58	8.57	8.56	8.55	8.54	8.54
4	5.77	5.75	5.73	5.72	5.70	5.69	5.68	5.66	5.65	5.65
5	4.52	4.50	4.48	4.46	4.44	4.43	4.42	4.41	4.39	4.39
6	3.83	3.81	3.79	3.77	3.75	3.74	3.73	3.71	3.70	3.69
7	3.40	3.38	3.36	3.34	3.32	3.30	3.29	3.27	3.26	3.25
8	3.11	3.08	3.06	3.04	3.02	3.01	2.99	2.97	2.96	2.95
9	2.89	2.86	2.84	2.83	2.80	2.79	2.77	2.76	2.74	2.73
10	2.73	2.70	2.68	2.66	2.64	2.62	2.60	2.59	2.57	2.56
11	2.60	2.57	2.55	2.53	2.51	2.49	2.47	2.46	2.44	2.43
12	2.50	2.47	2.44	2.43	2.40	2.38	2.37	2.35	2.33	2.32
13	2.41	2.38	2.36	2.34	2.31	2.30	2.28	2.26	2.24	2.23
14	2.34	2.31	2.28	2.27	2.24	2.22	2.21	2.19	2.17	2.16
15	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.10
16	2.23	2.19	2.17	2.15	2.12	2.11	2.09	2.07	2.05	2.04
17	2.18	2.15	2.12	2.10	2.08	2.06	2.04	2.02	2.00	1.99
18	2.14	2.11	2.08	2.06	2.04	2.02	2.00	1.98	1.96	1.95
19	2.11	2.07	2.05	2.03	2.00	1.98	1.96	1.94	1.92	1.91
20	2.07	2.04	2.01	1.99	1.97	1.95	1.93	1.91	1.89	1.88
21	2.05	2.01	1.98	1.96	1.94	1.92	1.90	1.88	1.86	1.84
22	2.02	1.98	1.96	1.94	1.91	1.89	1.87	1.85	1.83	1.82
23	2.00	1.96	1.93	1.91	1.88	1.86	1.84	1.82	1.80	1.79
24	1.97	1.94	1.91	1.89	1.86	1.84	1.82	1.80	1.78	1.77
25	1.96	1.92	1.89	1.87	1.84	1.82	1.80	1.78	1.76	1.75
26	1.94	1.90	1.87	1.85	1.82	1.80	1.78	1.76	1.74	1.73
27	1.92	1.88	1.86	1.84	1.81	1.79	1.76	1.74	1.72	1.71
28	1.91	1.87	1.84	1.82	1.79	1.77	1.75	1.73	1.70	1.69
29	1.89	1.85	1.83	1.81	1.77	1.75	1.73	1.71	1.69	1.67
30	1.88	1.84	1.81	1.79	1.76	1.74	1.72	1.70	1.67	1.66
35	1.82	1.79	1.76	1.74	1.70	1.68	1.66	1.63	1.61	1.60
40	1.78	1.74	1.72	1.69	1.66	1.64	1.61	1.59	1.56	1.55
50	1.73	1.69	1.66	1.63	1.60	1.58	1.55	1.52	1.50	1.48
60	1.69	1.65	1.62	1.59	1.56	1.53	1.51	1.48	1.45	1.44
70	1.66	1.62	1.59	1.57	1.53	1.50	1.48	1.45	1.42	1.40
80	1.64	1.60	1.57	1.54	1.51	1.48	1.45	1.43	1.39	1.38
90	1.63	1.59	1.55	1.53	1.49	1.46	1.44	1.41	1.38	1.36
100	1.62	1.57	1.54	1.52	1.48	1.45	1.42	1.39	1.36	1.34
120	1.60	1.55	1.52	1.50	1.46	1.43	1.40	1.37	1.33	1.32
150	1.58	1.54	1.50	1.48	1.44	1.41	1.38	1.34	1.31	1.29
200	1.56	1.52	1.48	1.46	1.41	1.39	1.35	1.32	1.28	1.26
250	1.55	1.50	1.47	1.44	1.40	1.37	1.34	1.31	1.27	1.25
300	1.54	1.50	1.46	1.43	1.39	1.36	1.33	1.30	1.26	1.23
400	1.53	1.49	1.45	1.42	1.38	1.35	1.32	1.28	1.24	1.22
500	1.53	1.48	1.45	1.42	1.38	1.35	1.31	1.28	1.23	1.21
600	1.52	1.48	1.44	1.41	1.37	1.34	1.31	1.27	1.23	1.20
750	1.52	1.47	1.44	1.41	1.37	1.34	1.30	1.26	1.22	1.20
1000	1.52	1.47	1.43	1.41	1.36	1.33	1.30	1.26	1.22	1.19