

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mechanical Engineering

Study Programme : Bachelor of Science Honours in Engineering
Name of the Examination : Final Examination
Course Code and Title : DMX6305- Modern Control Systems
Academic Year/Semester : 2023/2024
Date : 11th March 2025
Time : 09.30h - 12.30h
Duration : 3 hours

General instructions

1. Read all instructions carefully before answering the questions.
 2. This question paper consists of Five (5) questions on Five (5) pages.
 3. **Answer all questions.**
 4. All questions carry equal marks.
 5. This is a Closed Book Test (CBT).
 6. The answer for each question should commence from a new page.
 7. Answers should be in clear handwriting.
 8. Do not use a **RED** pen.
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Question 1

- a) Explain the concept of aliasing in digital control systems and how it can be prevented.
[5 marks]
- b) Describe the differences between an Analog-to-Digital Converter (ADC) and a Digital-to-Analog Converter (DAC).
[5 marks]
- c) What is the role of a digital filter in a digital control system? Provide an example.
[5 marks]
- d) Define the term 'sample and hold' in the context of digital control systems.
[5 marks]

Question 2

- a) Consider the open-loop sampled data system shown in Figure Q2. Determine the transfer function $G(z)$ when the sampling time is $T = 1$ s. [10 marks]

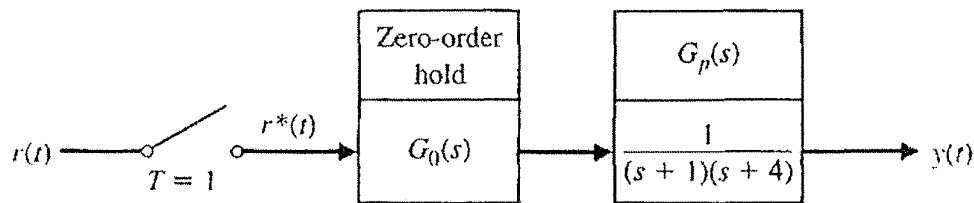


Figure Q2

- b) Given a continuous-time transfer function $H(s) = \frac{10}{s+5}$, convert this system to its discrete-time equivalent using a sampling period of $T = 0.1$ s. Use the bilinear transformation method. [10 marks]

Question 3

- a) Discuss the advantages of using state space over transfer function representation in digital control systems. [5 marks]
- b) Describe how the discrete-time state space model can be derived from a continuous-time state space model. [5 marks]
- c) Determine the state space representation's state transition matrix, control input matrix, and output matrix below. [5 marks]

$$\mathbf{x}(k+1) = \begin{bmatrix} 0.5 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.2 & 0.1 & 0.9 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}(k)$$

$$\mathbf{y}(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}(k)$$

- d) Define the characteristic equation of a discrete-time system and explain its significance in analyzing system stability. [5 marks]

Question 4

- a) Explain how the state transition matrix A_d affects the dynamic behavior of a discrete-time control system. [5 marks]
- b) Derive the discrete-time state transition matrix A_d for a sampling period $T = 0.1s$ below continuous-time system with state transition matrix. [5 marks]

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

- c) Explain the concept of the Linear Quadratic Regulator (LQR) in optimal control. [5 marks]
- d) Explain the concept of "Dynamic Programming" and how it relates to optimal control. [5 marks]

Question 5

You are required to design a digital control system for a first-order process with a time constant of 5 seconds and a gain of 2.

- a) Derive the continuous-time transfer function of the process. [10 marks]
- b) Discretize the transfer function using the bilinear approximation method with a sampling time of $T = 1s$. [10 marks]

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	—	—	$\delta_0(n-k)$ 1 $n = k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	1(t)	1(k)	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3 z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$1 - e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$	$e^{-akT} - e^{-bkT}$	$\frac{(e^{-aT} - e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Tze^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{[(aT-1+e^{-aT}) + (1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^3(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1} \sin \omega T}{1-2z^{-1} \cos \omega T + z^{-2}}$
15.	$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1} \cos \omega T}{1-2z^{-1} \cos \omega T + z^{-2}}$
16.	$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$	$e^{-akT} \sin \omega kT$	$\frac{e^{-aT} z^{-1} \sin \omega T}{1-2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
17.	$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$	$e^{-akT} \cos \omega kT$	$\frac{1-e^{-aT} z^{-1} \cos \omega T}{1-2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
18.	—	—	a^k	$\frac{1}{1-az^{-1}}$
19.	—	—	a^k $k = 1, 2, 3, \dots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	—	—	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	—	—	$k^2 a^{k-1}$	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	—	—	$k^3 a^{k-1}$	$\frac{z^{-1}(1+4az^{-1}+a^2 z^{-2})}{(1-az^{-1})^4}$
23.	—	—	$k^4 a^{k-1}$	$\frac{z^{-1}(1+11az^{-1}+11a^2 z^{-2}+a^3 z^{-3})}{(1-az^{-1})^5}$
24.	—	—	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$$x(t) = 0 \quad \text{for } t < 0$$

$$x(kT) = x(k) = 0 \quad \text{for } k < 0$$

Unless otherwise noted, $k = 0, 1, 2, 3, \dots$

Definition of the Z-transform

$$\mathcal{Z}\{x(k)\} = X(z) = \sum_{k=0}^{\infty} x(k)z^{-k}$$

Important properties and theorems of the Z-transform

	$x(t)$ or $x(k)$	$Z\{x(t)\}$ or $Z\{x(k)\}$
1.	$ax(t)$	$aX(z)$
2.	$ax_1(t) + bx_2(t)$	$aX_1(z) + bX_2(z)$
3.	$x(t+T)$ or $x(k+1)$	$zX(z) - zx(0)$
4.	$x(t+2T)$	$z^2X(z) - z^2x(0) - zx(T)$
5.	$x(k+2)$	$z^2X(z) - z^2x(0) - zx(1)$
6.	$x(t+kT)$	$z^kX(z) - z^kx(0) - z^{k-1}x(T) - \dots - zx(kT-T)$
7.	$x(t-kT)$	$z^{-k}X(z)$
8.	$x(n+k)$	$z^kX(z) - z^kx(0) - z^{k-1}x(1) - \dots - zx(k-1)$
9.	$x(n-k)$	$z^{-k}X(z)$
10.	$tx(t)$	$-Tz \frac{d}{dz} X(z)$
11.	$kx(k)$	$-z \frac{d}{dz} X(z)$
12.	$e^{-at}x(t)$	$X(ze^{aT})$
13.	$e^{-ak}x(k)$	$X(ze^a)$
14.	$a^kx(k)$	$X\left(\frac{z}{a}\right)$
15.	$ka^kx(k)$	$-z \frac{d}{dz} X\left(\frac{z}{a}\right)$
16.	$x(0)$	$\lim_{z \rightarrow \infty} X(z)$ if the limit exists
17.	$x(\infty)$	$\lim_{z \rightarrow 1} [(1-z^{-1})X(z)]$ if $(1-z^{-1})X(z)$ is analytic on and outside the unit circle
18.	$\nabla x(k) = x(k) - x(k-1)$	$(1-z^{-1})X(z)$
19.	$\Delta x(k) = x(k+1) - x(k)$	$(z-1)X(z) - zx(0)$
20.	$\sum_{k=0}^n x(k)$	$\frac{1}{1-z^{-1}}X(z)$
21.	$\frac{\partial}{\partial a} x(t, a)$	$\frac{\partial}{\partial a} X(z, a)$
22.	$k^m x(k)$	$\left(-z \frac{d}{dz}\right)^m X(z)$
23.	$\sum_{k=0}^n x(kT)y(nT-kT)$	$X(z)Y(z)$
24.	$\sum_{k=0}^{\infty} x(k)$	$X(1)$

END