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The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical and Computer Engineering

Study Programme	: Bachelor of Science Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: **EEX6441 Electromagnetism and Wave Propagation**
Academic Year	: 2023/2024
Date	: 09th Sunday, February 2025
Time	: 09:30 - 12:30 hrs
Duration	: **03 hours**

General Instructions

1. Read all the instructions carefully before answering the questions.
2. This is a Closed Book Test (CBT).
3. This question paper consists of **Five (05)** questions in **Three (03)** pages.
4. Answer **All five (05) questions**.
5. All questions carry equal marks.
6. The answer for each question should commence from a new page.
7. Answers should be in clear handwriting.
8. Do not use a Red color pen.
9. All notations have their usual meaning.
10. Assume any missing parameters with suitable values.

Q1.

(a) Derive the Poisson's equation from Gauss law.

[05 Marks]

(b) When four identical charges with charge Q are placed at four corners with side lengths of a, determine the stored energy W in the system.

[07 Marks]

(c) Determine **E** everywhere due to a spherically symmetric charge distribution with a charge density

$$\rho_v = \begin{cases} 10r & 0 \le r \le a \\ 0 & r > a \end{cases}$$

where a is the radius of a sphere.

[08 Marks]

Q2.

(a) State the Ampere's circuit law.

[04 Marks]

(b) Determine the magnetic flux through a rectangular loop $(a \times b)$ due to an infinitely long conductor carrying a current I as shown in Fig. Q2. The loop and the straight conductor are separated by a distance d.

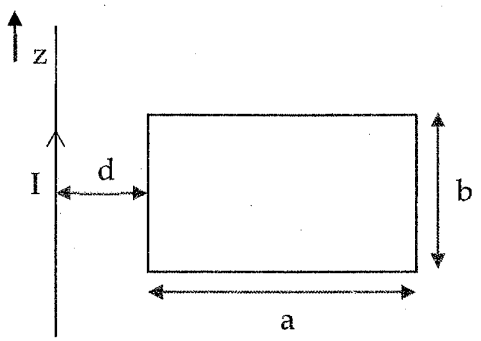


Fig. Q2

[06 Marks]

(c) A hollow conducting cylinder has an inner radius a and an outer radius b. If it carries a current I along positive z-direction, determine **H** everywhere.

[10 Marks]

Q3.

(a) Find the potential and electric field intensity for the region between two concentric right cylinders, where $V = 0$ at inner cylinder radius r_a and $V = V_0$ at outer cylinder radius r_b. Neglect the fringing.

[08 Marks]

(b) The electric field and magnetic field in free space are given by

$$\mathbf{E} = \frac{50}{\rho}\cos(10^6 t + \beta z)\mathbf{a}_\phi \quad \text{V/m}$$

$$\mathbf{H} = \frac{H_0}{\rho}\cos(10^6 t + \beta z)\mathbf{a}_\rho \quad \text{A/m}$$

Express those in phasor form and determine the constants H_0 and β such that the fields satisfy Maxwell's equations.

[12 Marks]

Q4.

(a) Compare and contrast the properties of different media used for wave propagation.

[04 Marks]

(b) State the Poynting theorem and explain its significance in wave propagation.

[04 Marks]

(c) A uniform plane wave propagating through a medium is defined by

$$\mathbf{E} = 2e^{-\alpha z}\sin(10^8 t - \beta z)\,\mathbf{a}_y \quad \text{V/m}$$

If the medium is characterized by $\varepsilon_r = 1$, $\mu_r = 20$ and $\sigma = 3$ mho/m, determine α, β and $\mathbf{H}$.

[12 Marks]

Q5.

(a) If the mode of propagation of a rectangular waveguide is TE_{11}, draw its field patterns.

[05 Marks]

P.T.O.

(b) The magnetic field component of a rectangular waveguide ($a = 1.5$ cm, $b = 0.8$ cm) operating in a dielectric medium with $\sigma = 0$ and $\varepsilon = 4\varepsilon_0$ is given by

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin(\pi \times 10^{11}t - \beta z) \quad \text{A/m}$$

Determine the following.

(i) Mode of operation
(ii) Cut-off frequency
(iii) Phase constant β
(iv) Propagation constant γ
(v) Intrinsic wave impedance η

[5 × 03 Marks]

Note:

1. **Cylindrical Coordinates**

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$$

$$\nabla \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{\boldsymbol{z}}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

2. **Spherical Coordinates**

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}},$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$

$$= \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right)f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \varphi^2}f,$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi},$$

$$\nabla \times \mathbf{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial \varphi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\hat{\boldsymbol{\varphi}},$$