THE OPEN UNIVERSITY OF SRI LANKA

Faculty of Engineering Technology

Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering

237

Final Examination (2022/2023)

MHZ3551: Engineering Mathematics I

Date: 31st January 2024 (Wednesday) **Time: 0930 hrs. – 1230 hrs.**

Instructions:

- *Answer **five (05)** questions only.*
- *This paper consists of **eight (08)** questions.*
- *The number of pages in the paper is **four (04)**.*
- *All the symbols are in standard notation unless they are defined.*

Q1. (a). Use the method of **contradiction** to prove the following statement. 30%

$$\forall\, x, y, c, k \in \mathbb{R}\ \forall\, a, b \in \mathbb{R}^{+} \left(ax + by > c \Rightarrow \left(x > \frac{c-k}{a} \vee y > \frac{k}{b} \right) \right).$$

(b). Using the method of **proof by cases**, prove that if $a \in \mathbb{Z}^{+}$and a is not divisible by 3, then $a^2 + 6a + 8$ is divisible by 3. 40%

(c). Applying the method of **indirect proof**, prove that if $5mn + 8$ is an irrational number, then m is an irrational number **or** n is an irrational number. 30%

Q2. (a). Let A and B be any two subsets of the universal set U. Using the **laws of sets** prove that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$. 30%

(b). A relation σ is defined on $\mathbb{Z}^{+}$such that $\sigma = \{ (x, y) | y = n^2 x \text{ for some } n \in \mathbb{Z}^{+} \}$. Prove that σ is a partially ordered relation. 30%

(c). If $f(x) = x^2$ for $x \in (-3, 2)$ and $g(x) = 2x + 3$ for $x \in (-3, 3)$, then find the range of f, R_f and the range of g, R_g. Find the function $f \circ g$. 40%

Q3. (a). Let $A = \begin{bmatrix} 8 & 5 \\ -4 & -13 \end{bmatrix}$. λ_1 and λ_2 are the values of λ such that $det(A - \lambda I_2) = 0$, where $\lambda_1 > \lambda_2$ and I_2 is the unit matrix of order 2.

i). Find the values of λ_1 and λ_2. 15%

ii). $AU = \lambda_1 U$ and $AV = \lambda_2 V$ where, $U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$. Find the values of a_1, a_2, b_1 and b_2 such that $U = u_2 \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $V = v_1 \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. 30%

iii). If $P = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ and $E = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, then prove that $P^{-1}AP = E$. 25%

(b). Applying **raw operations and column operations**, find the solutions of x of the equation $\begin{vmatrix} x+2a & 3b & 5c \\ 2a & x+3b & 5c \\ 2a & 3b & x+5c \end{vmatrix} = 0$. 30%

Q4. (a). Let x_n be a sequence such that

$$x_n = \frac{an+b}{cn^2+dn+1} + \frac{an+b}{cn^2+dn+2} + \frac{an+b}{cn^2+dn+3} + \cdots\cdots + \frac{an+b}{cn^2+dn+n}$$

, where $a, b, c, d \in \mathbb{R}$ and $c \neq 0$.

i). Show that $\frac{an+b}{cn+d+1} < x_n < \frac{n(an+b)}{cn^2+dn+1}$. 25%

ii). By using sandwich rule deduce that $\lim_{n\to\infty} x_n = \frac{a}{c}$. 15%

(b). Use Cauchy's root test to show that $\sum_{n=1}^{\infty} \frac{n^{2023}}{2023^n}$ is convergent. 20%

(c). Show that $\lim_{(x,y)\to(0,0)} \frac{4xy^2}{5x^2+6y^4}$ does not exist. 20%

(d). Show that $\frac{1}{\lambda^2}\frac{\partial^2 f}{\partial x^2} + \frac{1}{\mu^2}\frac{\partial^2 f}{\partial y^2} = 0$ if $f(x,y) = e^{-\lambda x}cos(\theta + \mu y)$, where λ, θ and μ are independent of x and y. 20%

Q5. **(a).** Let f be a function such that $f(x) = ln\left(\frac{1+x}{1-x}\right)$, where $|x| < 1$.

i). By using the mean value theorem for f prove that there exits $c \in (0, a)$ such that $\frac{2}{1-c^2} = \frac{1}{a} ln\left(\frac{1+a}{1-a}\right)$, where $a \in (0, 1)$. 25%

ii). Hence, prove that $2a < ln\left(\frac{1+a}{1-a}\right) < \frac{2a}{1-a^2}$. 25%

(b). Let f be a function such that $f(x) = x(x-2a)e^{\frac{x}{a}}$, where $a \in \mathbb{R}^+$. Show that 50%

$$\frac{df(x)}{dx} = \frac{1}{a}(x-\sqrt{2}a)(x+\sqrt{2}a)e^{\frac{x}{a}}.$$

Verify Roller's theorem for the above function f.

Q6. **(a).** Using the substitution $y = vx$, find the general solution of the following differential equation. 30%

$$4xcos\left(\frac{y}{x}\right)\frac{dy}{dx} = 4ycos\left(\frac{y}{x}\right) + 7x^8.$$

(b). Find the general solution of the following differential equation. 30%

$$\frac{dy}{dx} - 3y = 8e^{3x}\cos(9x + 45^0).$$

(c). Show that the following differential equation is exact and find its general solution. $(9x^2 - 8xy + 18y^2)dy + (24x^2 + 18xy - 4y^2)dx = 0$. 40%

Q7. **(a).** Let $y_P = ke^{6x}$ be a particular solution of the following differential equation,

$$\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 5e^{6x}.$$

i). Find the value of k. 15%

ii). Obtain the general solution of the above differential equation. 20%

iii). When $x = 0, y = 3$ and $\left(\frac{dy}{dx}\right)_{x=0} = 23$, then find the solution of the differential equation. 15%

(b). Using the D −operator method, find a particular solution of the differential equation $\frac{d^2y}{dx^2} - 121y = 185sin8x$. 50%

Hence, obtain the general solution of the given differential equation.

Q8. **(a).** A particle of mass m is projected vertically upward with the initial velocity u in a resistive medium with a resistance mkv^2, where v is the speed of the particle and k is a constant.

i). Prove that the greatest height reached by the particle is 20%

$$\frac{1}{2k} ln\left(1 + \frac{ku^2}{g}\right).$$

ii). Show also that the particle returns to the point of projection with the speed of $\dfrac{u}{\sqrt{1 + \dfrac{ku^2}{g}}}$. 30%

(b) A uniform rod of mass m and length $2a$ can turn freely about one end which is fixed. It is given an angular velocity ω from the position in which it hangs vertically.

i). Show that if θ is the inclination of the rod to the downward vertical at time t, then $\left(\dfrac{d\theta}{dt}\right)^2 = \omega^2 - \dfrac{3g}{2a}(1 - cos\theta)$. 20%

ii). Find the least value ω_0 of ω so that the rod may just make complete revolutions. For the angular velocity ω_0, deduce that 30%

$$\frac{d\theta}{dt} = \sqrt{\frac{3g}{a}} cos\left(\frac{\theta}{2}\right).$$

End

-Copyright reserved -