THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering
Bachelor of Technology Honors in Engineering

Final Examination (2022/2023)
MHZ3552: Engineering Mathematics II

Date: 06th February 2024 (Tuesday) **Time: 09:30 – 12:30**

Instruction:

- **There are four sections in this paper.**
- **Please answer a total of five (05) questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

SECTION – A

Q1.

a. A factory utilizes two machines, namely "*Machine A*" and "*Machine B*", to produce metal rods. The following table summarizes two samples of 1000 rods which are produced from each machine according to the size of the rods as small, Medium, and Large.

Size	*Number of rods from Mahine A*	*Number of rods from Machine B*
Small	350	320
Medium	430	360
Large	220	320

i. Create a suitable graph/chart to discuss the composition of the rods production from "Machine A" according to the sizes of the rods. Interpret your findings appropriately. **(15%)**

ii. Create an appropriate graph/chart to compare the production from "Machine A" and "Machine B." Interpret your findings in a relevant way. **(15%)**

b. A survey is done to identify the number of text messages sent on the previous day by each employee in an engineering company. The outcomes are depicted in the following stem and leaf diagram.

0	0	0	1	1	3	5	7	7	7	8	8
1	0	1	2	3	3	4	4	6	9		
2	0	1	3	3	7						
3	5	7									
4	0	1									
5	8										

(You may use $\sum x = 494$ and $\sum x^2 = 13{,}890.$)

i. Find the measures of central tendency (Mean, Mode, Median) for the number of Text messages were sent by each employee on the previous day. **(20%)**

ii. Find the range, interquartile range, and standard deviation of the sample data. **(20%)**

iii. Identify the nature of skewness shown by the distribution, justifying your answer. **(05%)**

iv. Construct a box and whisker plot to represent the given data, clearly indicating the presence of any outliers. **(15%)**

v. State, giving a reason, which measure of central tendency and measure of dispersion you would recommend to summarize these data. **(10%)**

Q2.

a. Saman knows that he has 3 tins of barium, 2 tins of bolts, and 2 tins of nuts in his garage. All the tins are identical in terms of their shape and dimensions, except the labels have been completely removed. Because of this, Saman is unaware of the contents of each tin. Saman requires barium for his work and proceeds to sequentially open the tins, selected at random until he finds a tin containing barium. Let the random variable, X denote the number of tins that he must open.

i. Find the value $P[X = 3]$. **(05%)**

ii. Draw up a table express the probability mass function for the random variable X. **(15%)**

iii. Find $E[X]$ and $V[X]$. **(10%)**

b. Cement is sold in bags containing a stated weight of 50 kg. However, the actual weight of cement in each bag not always 50kg and it is normally distribution with mean 48kg and standard deviation 2kg.

i. Find the probability of a bag containing cement less than the stated weight. **(25%)**

ii. In a stock of 30 bags, find the number of bags containing cement less than the stated weight. **(25%)**

iii. The mean weight of cement is changed so that 2% of the bags contain cement less than the stated weight. The standard deviation stays the same. Find the new mean weight of the cement. **(20%)**

SECTION – B

Q3.

a. Let $\boldsymbol{i} - 3\alpha\boldsymbol{j} + 2\alpha\boldsymbol{k}$ and $\alpha\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ be the position vectors of two points A and B respectively, with respect to a fixed origin O, where $\alpha \in \mathbb{R}^+$. Show that $A\hat{O}B = \frac{\pi}{2}$. **(25%)**

b. Prove that $[\boldsymbol{a} + \boldsymbol{b}, \boldsymbol{b} + \boldsymbol{c}, \boldsymbol{c} + \boldsymbol{a}] = 0$ if and only if $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ are coplanar vectors. **(35%)**

c. Find the equations of the lines L_1 and L_2 passing through points with position vectors, $\boldsymbol{a_1} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{b_1} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{a_2} = -\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{b_2} = 3\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ respectively. Hence show that the lines L_1 and L_2 are intersect each other. **(40%)**

Q4.

a. Find the sum of the three cubic roots of unity. **(30%)**

b. Find the values of $(-8)^{\frac{1}{3}}$. **(30%)**

c. Let $z \in \mathbb{C}$. Prove that,

$$\tan\left(i\, log\left(\frac{\bar{z}}{z}\right)\right) = -\frac{z^2 - \bar{z}^2}{z^2 + \bar{z}^2}.$$

[Hint: Use $z = re^{i\theta}$, where r and θ be the usual notations of a complex number]. **(40%)**

SECTION – C

Q5.

a. Show that the positive root of square root of 3 lies between 1 and 2. **(05%)**

b. An iterative formula for finding a square root of any positive real number a is given as follows,

$$x_{n+1} = \frac{x_n}{2}\left(1 + \frac{a}{x_n^2}\right);\ x_n \neq 0$$

i. Show that the above iterative formula is consistent for finding a square root of any positive real number a. **(15%)**

ii. Show that the above iterative formula is convergent to find the square root of 3. **(15%)**

iii. If the above iterative formula is convergent, then find the square root of 3, correct to two decimal places. **(25%)**

c. Let consider the following system of liner equations.

$$6x + y + z = 105$$
$$4x + 8y + 3z = 155$$
$$5x + 4y - 10z = 65$$

i. To find the solution for the above system of linear equations, set up the Gauss-Seidel iteration scheme. **(15%)**

ii. Iterate your scheme three times using the initial approximation as $x = y = z = 0$. Hence find the approximate solutions of the above system of equations. **(25%)**

Q6.

a. Find the Laplace transformation of the following functions of t

i. $f(t) = t + 2t^2 + 3t^5$ **(15%)**

ii. $f(t) = e^{-t}\sin 2t$ **(15%)**

b. Find the inverse Laplace transform of the function:

$$f(s) = \frac{s+4}{s(s-1)(s^2+4)}$$

(30%)

c. Use the Laplace transformation to find the solution of the following initial value problem.

$$\frac{d^2y}{dt^2} + y = \sin 3t$$

subject to the initial conditions $y(0) = 0,\ y'(0) = 0.$ **(40%)**

SECTION – D

Q7.

a.

i. Define the radius of convergence. **(15%)**

ii. Find the radius of convergence of the following series.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$$

(25%)

b. Find the McLaurin series for $f(x) = \frac{1}{1+x}$. Hence deduce the Taylor series expansions of the following functions.

i. $f(x) = \frac{1}{x+2}$; about $x = 1$

ii. $f(x) = \frac{1}{1+x^2}$; about $x = 0$

iii. $f(x) = \tan^{-1} x$; about $x = 0$

(60%)

Q8.

a. Show that an even function can have no ***sine*** terms in its Fourier series expansion. **(25%)**

b. A function is defined as,

$$f(x) = x - \pi \qquad ; -\pi < x < \pi$$

$f(x) = f(x + 2k\pi)$, where $k \in \mathbb{Z}$.

i. Draw a graph of f in the interval $[-3\pi, 3\pi]$. **(25%)**

ii. Find the Fourier series expansion of $f(x)$. **(25%)**

iii. Hence deduce that,

$$\frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)}$$

(25%)

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