THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering
Bachelor of Technology Honors in Engineering
Final Examination (2022/2023)
MHZ4553: Engineering Mathematics III

Date: 12^{th} February 2024 **Time: 09:30 a.m.-12:30 p.m.**

Instructions:

- This paper consists of EIGHT (8) questions in FOUR (4) pages.
- Answer any FIVE (5) questions ONLY. All questions carry equal marks.
- Answer for each question should commence from a new page.
- Relevant Statistic Tables are provided and all symbols are in standard notation.
- State any assumptions you required and show all your workings.
- This is a closed book test and do not use red color pen.

1. (a) Find $\dfrac{\partial z}{\partial y}$ assuming $\sqrt{xz} + \ln y = xyz$ defines z implicitly as a differentiable function of x and y on some open set $S \subseteq \mathbb{R}^2$. *(Marks 40)*

   (b) Let a scalar potential on a surface is given by $f(x, y) = xy$ for $(x, y) \in \mathbb{R}^2$.

   i. Find the highest rate of change of the potential at point $(1, -1)$ and its direction. *(Marks 40)*

   ii. Find the normal field line(path line) of the vector field ∇f passing through $(1, -1)$. *(Marks 20)*

2. Let $f(x, y) = e^y \sin x$ on $\mathbb{R}^2$.

   (a) Use the Taylor series approximation to find the quadratic approximation $Q(x, y)$, for f at the origin. *(marks 30)*

   (b) Find a linear function $L(x, y)$ and a symmetric matrix A such that $Q(x, y) = L(x, y) + \dfrac{1}{2}(x, y)A\begin{pmatrix} x \\ y \end{pmatrix}$. *(marks 20)*

   (c) Use the diagonalization of matrix A to change the variables (x, y) to (X, Y) and hence to find the real values a and b such that $\dfrac{1}{2}(x, y)A\begin{pmatrix} x \\ y \end{pmatrix} = aX^2 + bY^2$. *(marks 50)*

3. (a) Let $f(z) = \begin{cases} \dfrac{Re(z)}{z} & \text{if } z = 0 \\ 0 & \text{if } z \neq 0 \end{cases}$ on $\mathbb{C}$. Discuss the continuity of f on $\mathbb{C}$. Justify your answer. *(marks 20)*

(b) Let $u(x, y) = -y - 2xy$ be a Harmonic function defined on $\mathbb{R}^2$. Find a Harmonic conjugate of u. *(marks 40)*

(c) Compute $\int_C \dfrac{1}{(z^2+4)^2} dz$, where $C : |z - i| = \dfrac{3}{2}$. *(marks 40)*

4. Consider the following equation.

$$3x\frac{d^2y}{dx^2} + (x+2)\frac{dy}{dx} + y = 0 \tag{1}$$

(a) Find regular singular points of the equation (1). Justify your answer. *(Marks 20)*

(b) Find the series solution y of equation (1) around $x = 0$ with $y(0) = 1$. *(Marks 80)*

5. (a) Use the method of characteristics to find the solution of $u_x + e^{-2x}u_y = y$ subject to the initial condition $u(0, y) = \dfrac{3}{4} + y$. *(Marks 50)*

(b) Consider the following PDE.

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2} \quad \text{on } 0 \leq x \leq 1,\ 0 \leq t \leq 0.5$$
$$u(0, t) = u(1, t) = 0 \text{ for each } 0 \leq t \leq 0.5$$
$$u(x, 0) = \frac{x(1-x)}{2} \text{ for each } 0 \leq x \leq 1.$$

Use the explicit scheme of finite difference method with step sizes $h = 0.2$ and $k = 0.1$ to find an approximate value for $u(0.2, 0.2)$ with respect to given initial and boundary conditions. *(Marks 50)*

Hint:
$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{h},$$
$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \frac{u_{i,j+1} - u_{i,j}}{k}$$

where h and k are step sizes of variables x and t with respectively.

6. (a) Let $\vec{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \vec{u}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \vec{v} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$ and $W \subset \mathbb{R}^4$ such that $W = span(\vec{u}_1, \vec{u}_2)$.

i. Find a basis for W consisting of two orthogonal vectors. (*Marks 30*)

ii. Does $\vec{v} \in W$? Justify your answer. (*Marks 30*)

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a transformation defined by $T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ x + 3y \\ y \end{pmatrix}$. Is T a linear transformation? Justify your answer. (*Marks 40*)

7. (a) Discuss about the Type I error and Type II error in Hypothesis testing. (*Marks 10*)

(b) In a statistical study of identifying factors affecting the failures in electrical components, the researcher concludes that if the electrical component received a high voltage which exceeds the 600V there will be high chance to obtain failure. Depending on this conclusion, a certain electrical development company is going to develop electrical components which will not receive more than 600V of voltage on average. To test whether the electrical components are not receiving the high voltage, the quality assurance team had tested 36 electrical components and found that the average voltage absorbed by the given components is 548V and standard deviation 150V.

i. State the corresponding null hypotheses and alternative hypothesis to be tested. Comment whether this is two tail test or one tail test. (*Marks 30*)

ii. Find the test statistics associated with this hypothesis testing. (*Marks 20*)

iii. If the analyst wants to test this hypothesis at 5% significant level, then what is the critical value/s associated with this test? Explain why you chose the corresponding table when finding critical value/s. (*Marks 20*)

iv. Test the hypothesis at 5% significant level and state your decision regarding the hypothesis. Explain your conclusion regarding the claim of the electrical company. (*Marks 20*)

8. A study was conducted to determine whether the number of project involvement has an impact on the monthly compensation of engineers in an engineering company. The table below provides the number of project involvement and monthly income (in thousand rupees) of 10 engineers.

Number of projects	13	16	22	2	8	26	19	20	1	4
Monthly compensation	260	330	360	160	260	370	330	360	160	190

(a) Identify the independent variable and the dependent variable. *(Marks 10)*

(b) Calculate the correlation coefficient for above data. Interpret your answer. *(Marks 30)*

(c) Draw the scatter plot for dependent variable against the independent variable. *(Marks 10)*

(d) Find the coefficient of the equation of the best fitted simple linear regression model of the above data. *(Marks 30)*

(e) Interpret the found regression coefficient in part (d). *(Marks 10)*

(f) Use the estimated line of regression to find the monthly income of an engineer who is involved in 10 projects. *(Marks 10)*

End.

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Table entry for z is the area under the standard normal curve to the left of z.

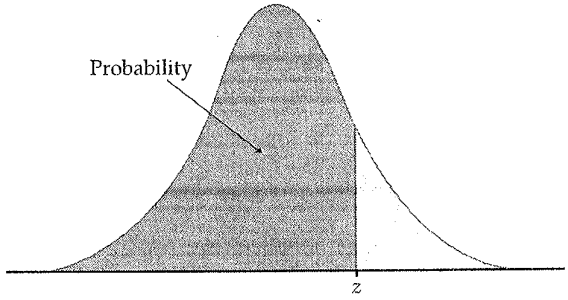


TABLE A

Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

t Distribution: Critical Values of *t*

Degrees of freedom	*Two-tailed test:* / *One-tailed test:*	*Significance level* 10% / 5%	5% / 2.5%	2% / 1%	1% / 0.5%	0.2% / 0.1%	0.1% / 0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291