

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering



Study Programme	: Bachelor of Science Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: CVX5242 Fluid Mechanics II
Academic Year	: 2022/2023
Date	: 11 th February 2024
Time	: 1330-1630 hrs
Duration	: 03 hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **SIX (06)** questions on **Five (05)** pages.
3. Answer **Any FIVE (05)** questions.
4. Answer for each question should commence from a new page.
5. Necessary additional information is provided.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear handwriting.
8. Do not use a Red colour pen.
9. Students are expected to know the standard denominations used for equations

Question (1)

a) For an incompressible flow, the fluid velocity components in three dimensional space are described by:

$$u = a(x^{1/2} - y^{1/2}) , v = -\frac{a}{2}x^{1/2}y , w = \frac{a}{2}y^{1/2}z$$

Show that the flow satisfies the Continuity Equation. (6 Marks)

b) Derive expression for the component of acceleration in x direction for the flow field described under Section (a). Format of the differential equation for the x component of acceleration is as follows. (6 Marks)

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y}$$

c) Investigate whether the flow field described under Section (a) is an exact solution for Navier Stokes Equations. Consider body forces x, y and z are equal to zero. (8 Marks)

Question (2)

a) Two incompressible viscous fluids flow on top of the other down an inclined plate at an angle α . They both have the same density ρ , but different viscosities μ_1 and μ_2 . The lower fluid has depth of h and the depth of upper flow is $2h$. Assuming that viscous forces from the surrounding air is negligible and the pressure on the free surface is constant, Show that: (10 Marks)

$$u_1(y) = \left[(h_1 + h_2)y - \frac{1}{2}y^2 \right] \frac{g \sin \alpha}{\nu_1}$$

b) Find the convective acceleration at the middle of a pipe which converges uniformly from 0.4m diameter to 0.2m diameter over a 2m length. The rate of flow is $0.02\text{m}^3\text{s}^{-1}$. If the rate of flow changes uniformly from $0.02\text{m}^3\text{s}^{-1}$ to $0.04\text{m}^3\text{s}^{-1}$ in 30 seconds, find the total acceleration at the middle of the pipe at 15th second. (10 marks)

Question (3)

a) Using Navier Stokes equations, derive the following two simplified equations for a steady, laminar and two-dimensional incompressible flow. (10 Marks)

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

b) Based on the simplified equations obtained in Section (a), derive the following two boundary layer equations for a steady, laminar and two-dimensional incompressible flow moving over a flat thin plate. You may make necessary assumptions for the derivation. (10 Marks)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{dp}{dy} = 0$$

Question (4)

Assuming that the gradient of the velocity profiles with respect to x can be neglected as compared with respect to y , the asymptotic velocity profile for a laminar boundary layer on a porous flat plate (with zero pressure gradient) with suction could be obtained.

(a) Show that the Prandtl's boundary layer equation reduces to,

$$v \frac{du}{dx} = \nu \frac{d^2 u}{dy^2}$$

and

$$\frac{dv}{dy} = 0$$

With boundary conditions,

$$y=0; \quad u=0$$

$$y=\delta; \quad u=U$$

(8 Marks)

(b) Introducing suction, $v = -V_w$, show that the solution for Prandtl's equation is given by,

(12 Marks)

$$\frac{u}{U} = 1 - e^{-\frac{V_w}{\nu} y}$$

Question (5)

a) Using the diagrams of flow profiles (Figure 5.1) and the partial derivative $(\frac{\partial u}{\partial y})$, explain the occurrence of boundary layer separation for a steady, two-dimensional incompressible laminar flow. (8 Marks)

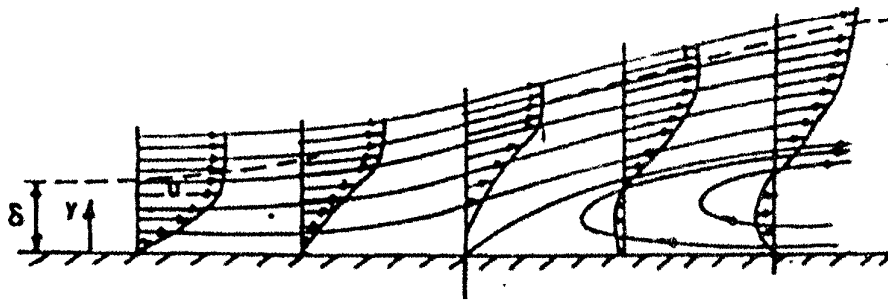


Figure 5.1

b) Derive the following equation from the Prandtl's boundary layer equation at the surface where $u = 0$. Using this equation, explain how the curvature of the velocity profile can be utilized to explain the occurrence of boundary layer separation. You may use the velocity profile diagrams for your explanations. (12 Marks)

$$\frac{dp}{dx} = \mu \left(\frac{\partial^2 u}{\partial y^2} \right)_0$$

Question (6)

Figure 6.1 shows a semi-finite plate positioned within a parallel flow. The flow is not subjected to any pressure gradient.

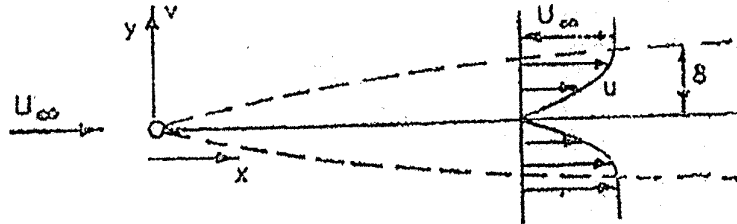


Figure 6.1

a) Using boundary layer equation derive the following expression

(5 Marks)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

b) The simplified boundary layer equation obtained in the above question 5 (a) can be solved to obtain simplified equation of motion for boundary layer by employing subjected approximations. Using appropriate approximations, drive the following non-linear third-order differential equation. (7 Marks)

$$f(\eta) f''(\eta) + 2f'''(\eta) = 0$$

You may use

$$\delta \propto \sqrt{\nu x / U_{\infty}}$$

$$\eta = y \sqrt{\frac{U_{\infty}}{\nu x}} = \frac{y}{\delta}$$

$$\psi = \sqrt{\nu x U_{\infty}} f(\eta)$$

$$u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}$$

c) Use the numerical results of Howarth given in Table 6.1, evaluate the boundary layer thickness δ , Displacement thickness δ^* , and momentum thickness θ for laminar boundary layer on a flat plate. (8 Marks)

Table 6.1: The function η for the laminar boundary layer along a flat plate at zero incidence (After L. Howarth)

$\eta = y \sqrt{\frac{U_\infty}{\nu x}}$	f	$f' = \frac{u}{U_\infty}$	f''
0	0	0	0.33206
0.4	0.02656	0.13277	0.33147
1.0	0.16557	0.32979	0.32301
1.4	0.32298	0.45627	0.30787
2.0	0.65003	0.62977	0.26675
2.4	0.92230	0.72899	0.22809
3.0	1.39682	0.84605	0.16136
3.4	1.74696	0.90177	0.11788
4.0	2.30576	0.95552	0.06424
4.4	2.69238	0.97587	0.03897
5.0	3.28329	0.99155	0.01591
5.4	3.68094	0.99616	0.00793
6.0	4.27964	0.99898	0.00240
6.4	4.67938	0.99961	0.00098
7.0	5.27926	0.99992	0.00022
7.4	5.67924	0.99998	0.00007
8.0	6.27923	1.00000	0.00001
8.4	6.67923	1.00000	0.00000