

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical & Computer Engineering

038



Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: EEX7434 Digital Signal Processing
Academic Year	: 2022/23
Date	: 10 th February 2024
Time	: 13.30-16.30hrs
Duration	: 3 hours

General Instructions

1. Read all instructions carefully before answering the questions.
 2. This question paper consists of **Five (5)** questions in **Four (4)** pages.
 3. Answer all questions. All questions carry equal marks.
 4. Answer for each question should commence from a new page.
 5. This is a Closed Book Test (CBT).
 6. Answers should be in clear hand writing.
 7. Do not use Red colour pens.
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Question 01 (20marks)

- a) Define Discrete Time Fourier Transform (DTFT) for the sequence $x[n]$. (2marks)
- b) If the DTFT of a signal $x[n]$ is known explain how to find $x[n]$. (2marks)
- c) Define the z-transform of a discrete-time sequence $x[n]$. What is the region of convergence (ROC)? (2marks)
- d) If the z-transform of $x[n]$ is $X[z]$, derive an expressions from first principles for the z-transform of $y[n]$ in terms $X[z]$ for the following: (8marks)
- (i) $y[n] = x[n - n_0]$
- (ii) $y[n] = a^n x[n]$
- e) Find z-transform of $u[n]$ from first principles. (2marks)
- f) Find the inverse z-transform of $X(z) = \frac{z^{-1}}{2-3z^{-1}+z^{-2}}$ for ROC $|z| > 1$. (4marks)

Question 02 (20marks)

- a) The continuous time signal $x(t) = \sin(20\pi t) + \cos(40\pi t)$ is sampled with a sampling period T to obtain the discrete-time signal. (4marks)

$$x[n] = \sin \frac{\pi n}{5} + \cos \frac{2\pi n}{5}$$

Determine a choice of T.

- b) A system is defined by the input output relation as, (4marks)
- $$y[n] = \cos [x(n)]$$
- Find whether the system is causal.

- c) A sinusoidal signal can be written as (4marks)

$$x[n] = A \cos \left(\frac{k}{L} n + \alpha \right)$$

where k and L do not have any common factors.

Prove that $x[n]$ is not periodic.

- d) Consider the following system, which computes the sum of a signal from time 0 up until time n:

$$y[n] = T\{x[n]\} = \sum_{k=0}^n x[k]$$

Where $x[n]$ is the input and $y[n]$ is the output.

- I. Show that the system is linear. (3marks)
- II. Show that the system is not time invariant. (3marks)
- III. Is the system bounded input bounded output (BIBO) stable? (2marks)

Question 03 (20marks)

- a) The input $x[n]$ and the output $y[n]$ of a causal system is related by the following difference equation:

$$y[n] - \frac{3}{8}y[n-1] + \frac{1}{8}y[n-2] = x[n].$$

- Find the system function $H(z)$. (4marks)
- Find the impulse response $h[n]$. (4marks)

b)

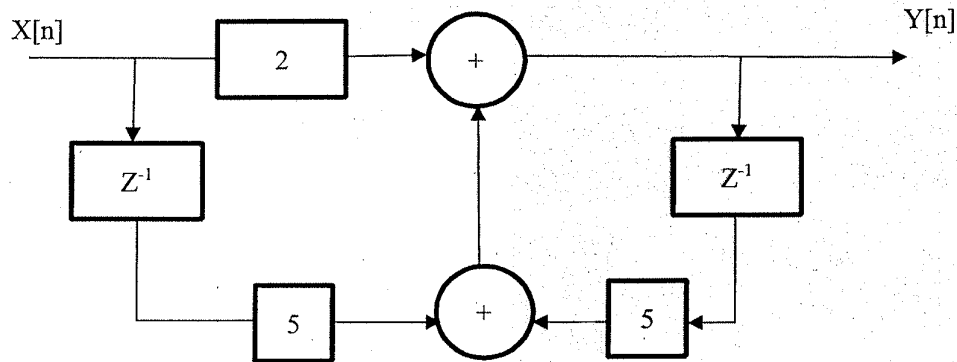


Figure 1

For the digital filter shown in Figure 1,

- Write state space equations. (6marks)
- Find the frequency response $H(e^{j\omega})$. (3marks)
- Find whether the filter is an FIR filter or an IIR filter. (3marks)

Question 04 (20marks)

a)

- Sketch the frequency response of a non-ideal lowpass filter and mark important regions. (4marks)
- Compare one critical parameter of a non-ideal filter with an ideal filter.

- b) It is necessary to design a low pass filter having a finite impulse response with following characteristics:

Pass band cutoff frequency = 7 kHz

Stop band cutoff frequency = 8 kHz with at least 50dB attenuation

Sampling Frequency = 40 kHz

- Convert pass band and stop band frequencies into digital frequencies. (2marks)
- Calculate the cutoff frequency ω_0 . (2marks)
- How can the causality of the filter be obtained? (2marks)
- Select a suitable window function to satisfy the requirements. (2marks)
- Find the length N of the filter. (4marks)
- Write an expression for the impulse response $h[n]$ of the filter. (4marks)

Question 05 (20marks)

Briefly explain the following:

- a) Upsampling and downsampling. (4marks)
- b) Realization of digital filters. (2marks)
- I. Realization by inspection of the difference equation (3marks)
- II. State space realization of an IIR filter (3marks)
- III. FIR filter realization using window functions (3marks)
- c) Kalman filtering. (4marks)
- d) Estimator model of a Kalman filter. (4marks)

Typical Window Functions

Window Type	$w(n)$	$\Delta\omega$	Attenuation
Rectangular	1	$\frac{4\pi}{N}$	-13 dB
Bartlett	$\frac{2}{N-1} \left(\frac{N-1}{2} - \left n - \frac{N-1}{2} \right \right)$	$\frac{8\pi}{N}$	-27 dB
Hanning	$0.5 + 0.5 \cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-32 dB
Hamming	$0.54 + 0.46 \cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-43 dB
Blackman	$0.42 + 0.5 \cos\left(\frac{2\pi n}{N-1}\right) + 0.5 + 0.08 \cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-53 dB

Z Transform Pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$\delta[n-m]$	z^{-m}	All z, except 0 (if $m > 0$) or ∞ (if $m < 0$)
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z > a $
$-a^n u[-n-1]$	$\frac{1}{1 - az^{-1}}$	$ z < a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z > a $

Sum of an infinite geometric series: $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$

Z Transform Formula: $X(z) = \sum x[n] \cdot z^{-n}$