

The Open University of Sri Lanka
 B.Sc./B.Ed. Degree Programme
 Pure Mathematics – Level 05
 PEU5302- Combinatorics
 NO BOOK TEST (NBT) – 2023/24



DURATION: ONE (01)–HOUR

Date: 03.09.2023.

Time :01.00 p.m.-02.00 p.m.

Answer all the Questions.

- (01) Let A and B be any two events in a probability space. Without using Venn diagram,
- show that $P(A \cap B^c) = P(A) - P(A \cap B)$, where B^c is the complement event of B .
 - If A and B are independent, deduce that $P(A \setminus B) = P(A)P(B^c)$.
- (02) Let n be a positive integer. Prove each of the following statements by the method of Proof by mathematical induction:
- $2 + 4 + 6 + \dots + 2n = n(n + 1)$,
 - $\left\{ \frac{1}{(3 \cdot 5)} \right\} + \left\{ \frac{1}{(5 \cdot 7)} \right\} + \left\{ \frac{1}{(7 \cdot 9)} \right\} + \dots + \left\{ \frac{1}{(2n+1)(2n+3)} \right\} = \frac{n}{3(2n+3)}$.
- (03) Let n and r be two non-negative integers such that $0 \leq r \leq n$. Let $c_r \left[\binom{n}{r} \right]$ be the coefficient of the $(r + 1)^{th}$ term of a binomial expansion of $(1 + x)^n$.
 Prove that $c_0 c_1 + c_1 c_2 + c_2 c_3 + \dots + c_{n-1} c_n = \frac{(2n)!}{(n+1)!(n-1)!}$.
- (04) (a) In the expansion of $(x - y - z + w)^{10}$, what is the multinomial coefficient of the term $x^2 y^3 z^4 w$?
- (b) Determine the coefficient of x^4 in the multinomial expansion of $(x - 2y + wx^2)^8$.