The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Pure Mathematics – Level 05
PEU5302- Combinatorics
NO BOOK TEST (NBT) – 2023/24

DURATION: ONE (01)–HOUR

Date: 03.09.2023. Time :01.00 p.m.-02.00 p.m.

Answer all the Questions.

(01) Let A and B be any two events in a probability space. Without using Venn diagram,

(i) show that $P(A \cap B^c) = P(A) - P(A \cap B)$, where B^c is the complement event of B.

(ii) If A and B are independent, deduce that $P(A\backslash B) = P(A)P(B^c)$.

(02) Let n be a positive integer. Prove each of the following statements by the method of Proof by mathematical induction:

(a) $2 + 4 + 6 + \cdots + 2n = n(n + 1)$,

(b) $\left\{\frac{1}{(3\cdot5)}\right\} + \left\{\frac{1}{(5\cdot7)}\right\} + \left\{\frac{1}{(7\cdot9)}\right\} + \cdots + \left\{\frac{1}{(2n+1)(2n+3)}\right\} = \frac{n}{\{3(2n+3)\}}$.

(03) Let n and r be two non- negative integers such that $0 \leq r \leq n$. Let $c_r\left[\binom{n}{r}\right]$ be the coefficient of the $(r + 1)^{th}$ term of a binomial expansion of $(1 + x)^n$.
Prove that $c_0c_1 + c_1c_2 + c_2c_3 + \cdots + c_{n-1}c_n = \frac{(2n)!}{(n+1)!(n-1)!}$

(04) (a) In the expansion of $(x - y - z + w)^{10}$, what is the multinomial coefficient of the term $x^2y^3z^4w$?

(b) Determine the coefficient of x^4 in the multinomial expansion of $(x - 2y + wx^2)^8$.