THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Bachelor of Software Engineering Honours Study Programme

Final Examination (2021/2022)
MHZ4256: Mathematics for Computing

Date: 23rd November 2022 (Wednesday) **Time: 09:30 hrs. - 12:30 hrs.**

Instructions:

- This paper consists of Nine (9) questions in Five (5) pages.
- Choosing at least one from each single section, answer total of **six (6)** questions only.
- State any assumptions you required.

Section A

1. (a) Decide which of the following are propositions. What are the truth values of those that are propositions? [20%]
   i. $\forall x \in \mathbb{R}^+$, if $x < -3$, then $x^3 = -1000$.
   ii. $\exists x \in \mathbb{Z}$, such that $5x^2 = 40$.
   iii. This statement is false.
   iv. The sine function is periodic and 2π is an integer.

   (b) State the **converse**, **inverse**, and **contrapositive** of each of the following statements: [30%]
   i. If a solution exists, then the program terminates.
   ii. If three points are on the same line, then they are collinear.

   (c) For each of the following formulas tell whether it is tautology or contradiction by using truth table. [20%]
   i. $(p \wedge q) \wedge \neg(p \vee q)$
   ii. $(p \rightarrow q) \longleftrightarrow (\neg p \vee q)$

   (d) Verify that,
   $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \equiv p \wedge q$ by using laws of the algebra of propositions. [10%]

   (e) Determine the truth values of the following statements and justify your answers by giving examples. Write down the negation of the following statements. [20%]
   i. $\forall x \in \mathbb{R}^+$, $\exists y \in \mathbb{R}^+$ such that $y^2 = x$.
   ii. $\exists n \in \mathbb{Z}^+$, such that $\forall m \in \mathbb{Z}, m^n > 0$

2. (a) Test the validity of the following argument? [25%]
We are given that if a program does not fail, then it begins and terminates, and we know that our program began and failed. We conclude that it did not terminate.

(b) i. Let n be an integer. Use a proof by cases to show that $n^2 - 3$ is not a multiple of 4. [10%]

ii. Suppose x be an integer [$x \in \mathbb{Z}$]. Use a proof by contraposition to show that if $7x + 9$ is even, then x is odd. [10%]

(c) Using Mathematical Induction, for a positive integer n, prove the following,
$2^2 + 4^2 + ... + (2n)^2 = \frac{2n(n+1)(2n+1)}{3}$ for all $n \geq 1$ [25%]

(d) By using truth tables, prove the following, [30%]
$p \vee (\neg p \wedge q) = p \vee q$

3. (a) Consider the following truth table. [35%]

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Where A, B and C are input variables and F is an output.

i. Create a Karnaugh map (K-map) based on the above truth table.

ii. Using the K-map created in the above part (a)(i), write down the Boolean function.

(b) Consider the following Boolean function given below. [35%]
$F = \overline{x}\,\overline{y}\,\overline{z} + \overline{x}\,\overline{y}\,z + \overline{x}\,y\,z + x\,\overline{y}\,z + x\,y\,z + x\,y\,\overline{z}$
Create a K-Map based on the above boolean function and minimized the above Boolean function using the K-Map.

(c) Minimize the following Boolean function by Algebraic method. [30%]

i. $F_1 = (\overline{A} + B)(\overline{A} + \overline{B})$

ii. $F_2 = A\,\overline{(A\,\overline{B} + \overline{A}\,C)} + (B\,\overline{C} + \overline{A}\,\overline{C})$

Section B

4. (a) Write down the elements in each of the following sets: [15%]

   i. $A = \{x : 4 + x = 3; x \in \mathbb{N}\}$

   ii. $B = \{x : |x - 2| \leq 5; x \in \mathbb{Z}^+\}$

   iii. $D = \{x : x^2 - 4 = 0; x \in \mathbb{Z}\}$

   (b) i. Define the Cartesian product of two sets. [10%]

   ii. If $A = \{1, 3, 5\}$ and $B = \{2, 3\}$, [10%]
   then compute $A \times B$ and $B \times A$.

   iii. If $A = \{(a, b), c\}$. List all elements of the power set $P(A)$. [15%]

   (c) Show that,
   $(B - A) \cup (C - A) = (B \cup C) - A$
   without using Venn Diagrams. [25%]

   (d) Suppose you have the sets,
   Universal set$(U) = \{x : x \in \mathbb{Z}; 1 \leq x < 20\}\}$
   $P = \{x : x$ is a prime number$, x \in \mathbb{N}$ and $x < 20\}$
   $Q = \{x : x = 2n + 1, n \in \mathbb{Z}$ and $0 \leq n, < 10\}$
   $R = \{x : 3|x; x \in \mathbb{Z}^+, x \leq 20\}$
   Find the elements of the following sets, [25%]

   i. $P \cap (Q - R)$

   ii. $(P \cup Q)' \cap R$

5. (a) Find the domain of each of the following real-valued functions: [20%]

   i. $f(x) = \dfrac{x^2 + 3x + 5}{x^2 - 5x + 4}$

   ii. $g(x) = \sqrt{16 - x^2}$

   (b) Let the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} 3x - 1, & \text{if } x > 3 \\ x^2 - 2, & \text{if } -2 \leq x \leq 3 \\ 2x + 3, & \text{if } x < -2 \end{cases}$$

   Calculate $f(3)$ and $f(-3)$. [20%]

   (c) Consider the following functions
   $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$, where $x \in \mathbb{R}$.
   Find a formula defining the composition function. [30%]

   i. $f \circ g\,(x)$

   ii. $g \circ f\,(x)$

(d) Let $A = \mathbb{R} - \left\{\frac{7}{5}\right\}$ and $B = \mathbb{R} - \left\{\frac{2}{5}\right\}$, and define $f(x) = \frac{2x-3}{5x-7}$ for $x \in A$. Then $f : A \longrightarrow B$. [30%]

i. Show that f is one-to-one.

ii. Show that f maps A onto B.

iii. Find the inverse of f.

6. (a) Let $A = \{5, 6, 7, 8, 9\}$ and $B = \{1, 2, 3, 4, 5\}$. In each of the following, find all the pairs of $A \times A$ and $B \times B$ that belong to R_1 and R_2.

i. $R_1 = \{(x, y) : x + 2y \leq 20;\ x, y \in A\}$ [10%]

ii. $R_2 = \{(x, y) : (x - y)$ is an even natural number; $x, y \in B\}$ [10%]

(b) i. Define the equivalence relation by the usual notation. [10%]

ii. Show that a relation R defined on the set of natural numbers $\mathbb{N}$ by $x\mathrm{R}y$ if and only if $(x.y) > 0$ for all $x, y \in \mathbb{N}$ is an equivalence relation. [30%]

(c) i. Define the partial order relation by using the usual notation. [10%]

ii. Show that the relation $x \leq y$ defined on the set of integers is a partial order relation. [30%]

Section C

7. (a) Express in the form $x + iy$. [30%]

i. $\frac{2+i}{3-4i}$

ii. $\frac{(5+4i)^2}{(2-3i)^2}$

(b) Verify the triangle inequality of the following complex numbers. [20%]
$z_1 = 2 + 3i, z_2 = 4 - i$.

(c) Find the modulus of the following complex numbers by reducing them to the form $a + bi$. [20%]

i. $(5 + 6i^{11}) + (8i^3 + i^5) + (i^2 - i^4)$

ii. $(\overline{7 + i^2}) + (6 - i) + (4 - 3i^3)$

(d) Express the following in the form $r\,(cos\,\theta + i\ sin\,\theta)$. [30%]

i. $1 - i$

ii. $\frac{2 + 6\sqrt{3}i}{5 + \sqrt{3}i}$

8. (a) If $z = x + iy$, then prove that $\left[\frac{z}{\overline{z}} + \frac{\overline{z}}{z}\right] = 2\left[\frac{x^2 - y^2}{x^2 + y^2}\right]$. [25%]

(b) Prove the following, for complex numbers z and w. [30%]

i. $|z + w|^2 = |z|^2 + |w|^2 + 2\,Re\,(z\,\overline{w})$.

ii. If $|z + w|^2 = |z|^2 + |w|^2$ then $\frac{z}{w}$ is purely imaginary.

(c) Let $p\,(z) = 3z^4 - 5z^3 + z^2 - 5z - 2$.

i. Show $p\,(-i) = 0$. [10%]

ii. Illustrate that $z^2 + 1$ is a factor of $p\,(z)$ (without division) and also find the other quadratic factor. [15%]

iii. Hence, find the linear factors of $p\,(z)$. [20%]

9. (a) Find the complex numbers representing the vertices of a square if one vertex represents $2 - 2i$ and the centre of the square is at the origin. [40%]

(b) Sketch and obtain the Cartesian form of the locus of z in each of the following cases.

i. $|z - 1 + 2i| = 4$ [15%]

ii. $arg(z + 2 - i) = \frac{\pi}{4}$ [15%]

(c) Shade the regions in the complex plane that satisfy the following inequalities. [30%]

i. $\frac{5\pi}{6} < arg(z) \leq \frac{\pi}{4}$

ii. $|z - 5 - 7i| < 3$

iii. $3 \leq |z - 1 + i| < 4$

End

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