THE OPEN UNIVERSITY OF SRI LANKA

Faculty of Engineering Technology

Department of Mathematics & Philosophy of Engineering

Bachelor of Software Engineering Honors

Final Examination (2024/2025)

MHZ4256: Mathematics for Computing

Date: 16th August 2025 (Saturday) **Time: 0930 hrs. – 1230 hrs.**

Instructions:

- *Answer **six (06)** questions by selecting at least **one question from each of sections A, B and C.***
- *This paper consists of **nine (09)** questions.*
- *All the symbols are in standard notation unless they are defined.*

Section – A

Q1. **a).** State whether each of the following statements is a proposition or not. Explain the truth value of each. 25%

i). $\forall x \in \mathbb{R}^+ \exists y \in \mathbb{R}^+ y < x.$

ii). $\forall x \in \mathbb{R}^+ \exists y \in \mathbb{R}^+ (x + y = z).$

b). Let p and q be propositions. 45%

i). Construct one truth table for the propositions $\neg p$, $p \vee q$, $(\neg p \Rightarrow q)$, $\neg p \wedge q$, $\neg p \vee q$ and $\neg p \Leftrightarrow q$.

α). Show that $\neg p \Leftrightarrow q$ is a logical consequence of $\neg p \Rightarrow q$, $\neg p \wedge q$ and $p \vee q$.

β). Show that $\big((\neg p \Rightarrow q) \wedge (\neg p \vee q)\big) \Rightarrow (\neg p \Leftrightarrow q)$ is a tautology.

γ). What is the relation between $\neg p \Rightarrow q$ and $p \vee q$?

c). Let p and q be propositions. By using laws of propositions, prove that $(\neg q \Rightarrow \neg p) \Rightarrow (p \Rightarrow q)$ is a tautology. 30%

Q2. **a).** Using the method of direct proof, show that any positive integer divisible by 4 can be written as a difference of two squares. 20%

b). Using the method of indirect proof, prove that if $n \in \mathbb{Z}^+$ and $6^n - 1$ is prime, then n is odd. 20%

c). Consider the following argument: 30%

All girls with wings are fairies.

Some girls who are not fairies have sharp nails.

Therefore, there are girls with no wings with sharp nails.

Taking the domain is all girls and defining the predicates as.

$W(x)$: x has wings, $F(x)$: x is a fairy, $S(x)$: x has sharp nails.

i). Symbolize the argument in quantification form.

ii). Check the validity of the argument.

d). Is the following equivalence valid? Justify your answer. 30%

$\forall x \exists y\, P(x,y) \equiv \exists y \forall x\, P(x,y)$. Give a counter example in mathematics.

Q3. **a).** Consider the statement " For every a and b rational numbers, and for every x and y real numbers, if $axy + b$ is irrational number, then x or y is irrational number." 30%

i). Translate the above statement into quantification form.

ii). Use the method of proof by a contradiction to prove the translated statement.

b). By using the principle of mathematical induction, prove that for all $n \in \mathbb{Z}^+$ 30%

$7^n - 1$ is divisible by 6.

c). Using the proof by case, show that if n is not divisible by 3, then $7n^2 + 5$ is divisible by 3. 40%

Section – B

Q4. **a).** Let x and y be Boolean variables. A Boolean function $f(x,y)$ satisfies the following truth table. 50%

x	y	$f(x,y)$
0	0	1
0	1	0
1	0	1
1	1	1

i). Find the corresponding Karnaugh map to the above truth table.

ii). Using the Karnaugh map, minimize the corresponding Boolean function.

b). Let x, y and z be Boolean variables. By using laws of Boolean algebra, show that 20%

$xy + \bar{x}z + yz = xy + \bar{x}z$.

c). Let x, y and z be Boolean variables. Using laws of Boolean algebra, minimize the following expressions. 30%

i). $F_1(x,y) = (\bar{x} + y)(\bar{x} + \bar{y})$.

ii). $F_2(x,y,z) = \overline{(x+y)}z + \bar{x}yz + x\bar{y}$.

Q5. **a).** Let the function $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be defined as follows: 20%

$$f(x) = \begin{cases} x^2 + 2x - 3, & \text{if} -\infty < x < -5 \\ x^2 + 2x + 4, & \text{if } 0 < |x| \leq 5 \\ x^2 - 2x + 3, & \text{if } 5 < x < \infty \end{cases}.$$

Find the values of $f(-3)$, $f(0), f(-6)$ and $f(6)$, if they are defined.

b). Let $A = \mathbb{R} - \left\{\frac{2}{7}\right\}$ and $B = \mathbb{R} - \left\{\frac{3}{2}\right\}$. The function f is defined as follows. 50%

$f: A \to B$ and $f(x) = \frac{3x + 5}{2x - 7}$.

i). Show that f is one to one function.

ii). Show that f is onto function.

iii). Find the inverse function of f.

c). The real valued functions f and g are defined as $f(x) = 3x + 2$ and $g(x) = \sin x$. Find $f \circ g$ and $g \circ f$. 30%

Q6. **a).** Let A, B and C be any subsets of a universal set S. 30%

i). Using laws of set theory prove that $A - (A - B) = A \cap B$.

ii). Give counter example to prove $P(A \cup B) \neq P(A) \cup P(B)$.

b). Let $A = \{0, 1, 2\}$ and $B = \{1, 2, 6\}$. 30%

i). Find $A \times B$ and $B \times A$.

ii). Let $R = \{(1, 1), (2, 2), (1, 2), (2, 1), (0, 2)\}$. Is R a relation from A to B or B to A? Justify your answer.

c). Let R be a relation defined on $\mathbb{N} \times \mathbb{N}$ as $(a, b)R(c, d)$, if and only if $4bd(a - c) = 5ac(d - b)$. Then show that R is an equivalent relation. 40%

Section – C

Q7. **a).** Prove that $3 + 4i = (2 + i)^2$, where $i^2 = -1$. 40%

Hence, solve the quadratic equation $x^2 + 3ix - 3 - i = 0$.

b). Find the values of $(1)^{\frac{1}{4}}$, that is, all complex solutions of the equation $z^4 = 1$. 30%

Let A and B be two sets such that $A = \left\{x \mid x = (1^2)^{\frac{1}{4}}\right\}$ and $B = \left\{x \mid x = \left((1)^{\frac{1}{4}}\right)^2\right\}$.

Show that B is a proper subset of A.

c). Write down a quadratic equation of the form $ax^2 + bx + c = 0$, satisfying the three conditions, where a, b, c are non-zero, $b^2 - 4ac \in \mathbb{R}^+$ and the roots are imaginary numbers. Justify your answer. 30%

Q8. **a).** Let $z, z_1, z_2 \in \mathbb{C}$. Prove the following results. 40%

i). $z\bar{z} = |z|^2$ **ii).** $\overline{z_1 + z_2} = \bar{z_1} + \bar{z_2}$ **iii).** $\overline{z_1 z_2} = \bar{z_1}\bar{z_2}$

vi). $z + \bar{z} = 2Re(z)$ **v).** $\bar{\bar{z}} = z$ **vi).** $Re(-z) = -Re(z)$

Using the above results, prove that $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z_2})$.

b). Prove that $(|z_1| + |z_2|)^2 - |z_1 + z_2|^2 = 2\big(|z_1||z_2| - Re(z_1\bar{z_2})\big)$. 30%

Deduce that $|z_1| + |z_2| \geq |z_1 + z_2|$.

c). Suppose that $|z_1 + z_2| = |z_1 - z_2|$. Prove that $\frac{z_1}{z_2}$ is purely imaginary. 30%

Let P_1 and P_2 be the points representing the above complex numbers z_1 and z_2 in an Argand (complex) plane. Show that P_1P_2 subtends an right angle at the origin.

Q9. **a).** Let $z = 1 - \cos\theta + i\sin\theta$, where $-\pi < \theta \leq \pi$. Find the $|z|$, $Arg(z)$ and $arg(z)$. 20%

b). Prove that the roots of the equation $z^3 = -125$ can be expressed as 30%

$5(\cos\pi + i\sin\pi), 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and $5\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$

c). Consider the following three paths in the Argand (complex) plane: 50%

P_1: $\text{Arg}(z + a) = \theta$,

P_2: $\text{Arg}(z - a) = \frac{\pi}{2} + \theta$,

P_3: $|z| = a$, where $a > 0$ and $0 < \theta < \frac{\pi}{2}$.

i). Sketch the curves P_1, P_2 and P_3 on the same Argand diagram.

ii). Find all complex numbers z that simultaneously satisfy both conditions of path P_1 and P_2.

iii). Shade the region of the Argand (complex) plane satisfying the following inequalities:

I_1: $\theta \leq \text{Arg}(z + a) \leq \frac{\pi}{2}$,

I_2: $\frac{\pi}{2} + \theta \leq \text{Arg}(z - a) \leq \pi$,

I_3: $|z| \leq a$, where $a > 0$ and $0 < \theta < \frac{\pi}{2}$.

End

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