THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Software Engineering Honors

Final Examination (2022/2023)
MHZ4256: Mathematics for Computing

Date: 24th May 2023 (Wednesday) **Time: 09:30 – 12:30**

Instruction:

- Answer only six questions.
- Please answer a total of six questions choosing at least one from each section.
- All symbols are in standard notations. State any assumption that you made.

SECTION – A

Q1.

I. Determine whether each of the following sentences is a proposition.

If the sentence is a proposition, then write down the truth value of the sentence. **[20%]**

a) "If $8 + 15 = 15$ then, $12 - 4 = 15$ and $7 - 3 = 4$" ;

b) "9 is a perfect square number but 23 is a prime number".

c) " $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, [x + y = 2]$";

d) " $x \leq 15$".

II. State the "converse", "inverse", and "contrapositive" of the following statement: **[15%]**

"If two sides of a triangle are equal, then the opposite angles are equal".

III. Let p, q and r be three statements.
Using a truth table, prove that the compound statement,
$[(p \rightarrow q) \wedge (r \rightarrow q)] \leftrightarrow [(p \vee r) \rightarrow q]$ statement is a tautology. **[20%]**

IV. Simplify the compound proposition $(q \wedge p) \vee \sim(q \rightarrow p)$ by using logical equivalences. **[25%]**

V. State the negation of the following statements: **[20%]**

a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ x - y^2 = 15$;

b) $\forall x \in \mathbb{R},\ \forall y \in \mathbb{R},\ \frac{x+y}{2} \geq \sqrt{xy}$.

Q2.

I. Test the validity of the following argument: **[25%]**

If there is cream, then I will drink coffee.
If there is a donut, then I will drink coffee.
There is no cream and there is a donut.

Therefore, I drink coffee.

II. If x is an integer, then prove that $4x^2 - 1$ is an odd integer. **[20%]**

III. Let n be a positive integer. Using the principle of mathematical induction. Prove that:

a) $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$ **[20%]**

b) $f(n) = 3^{2n} - 1$ is divisible by 8. **[20%]**

IV. Let x be an integer. If $13x + 5$ is even integer, then show that x is odd integer. **[15%]**

Q3.

I. Let p, q and r are input variables and F is an output. Consider the following truth table. **[25%]**

p	q	r	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

a) Create a Karnaugh map (K-map) based on the above truth table.

b) Using the created K-map in the above part I.(a), write down the Boolean function.

II. Consider the following Boolean function given below. **[30%]**

$$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} + ABC + A\bar{B}\bar{C} + AB\bar{C}$$

Create a K-map based on the above Boolean function and minimize the above Boolean function using a K-map.

III. Minimize the following Boolean functions by using the algebraic method. [45%]

a) $F_1 = (A + C)(AD + A\bar{D}) + AC + C$

b) $F_2 = A[B + C(AB + A\bar{C})]$

c) $F_3 = AB + \overline{AC} + A\bar{B}C(AB + C)$

SECTION – B

Q4.

I. Write down the elements in each of the following sets: [20%]

a) $A = \{x : x = 2n,\ n \in \mathbb{Z}^+,\ n < 8\}$,

b) $B = \{x: x = n^2 + 1, -2 \leq n < 3, n \in \mathbb{Z}\}$,

c) $C = \{x: x^2 + 1 = 0,\ x \text{ is a complex number}\}$,

d) $D = \{x : |x - 1| \leq 5, and\ x \in \mathbb{Z}^+\}$.

II. Let $U = \{1,2,3,4,5,6,7,8,9,10\}$, $M = \{x: x \text{ is an odd integer}, 1 < x < 10\}$, $N = \{x: x \text{ is an integer}, x^3 - 7x^2 + 14x - 8 = 0\}$ and $L = \{x: x = 3n - 1, 0 < n < 4, n \in \mathbb{Z}\}$. [20%]

a) Represent the above sets in a Venn diagram.

b) Find $L' \setminus (M \cup N)$ and $(M \setminus L) \cup (L \setminus N)$.

III.

a) Define the Cartesian product of two sets. [05%]

b) Let $A = \{1, 2, 3\}, B = \{2,4\}$ and $C = \{3, 4\}$.
Find $A \times B$ and $(A \times B) \times C$. [20%]

IV. Let $E = \{a,\ b, a + b,\ ab\}$. Find the power set of E. [15%]

V. Without using Venn diagrams, show that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. [20%]

Q5.

I. Find the domain of the following functions:

a) $f_1(x) = \frac{2x+3}{3x+5}$, [05%]

b) $f_2(x) = \frac{x-1}{\sqrt{(2-x)}}$, [10%]

c) $f_3(x) = \frac{\sqrt{x+2}}{(x^2-9)}$. [15%]

II. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x+7\ ; & x \leq 0 \\ -2x+5; & 0 < x < 3 \\ x-1; & x \geq 3 \end{cases}$$

Find $f(0), f(-10), f(2)$, and $f(3)$. [20%]

III. Let $g: \mathbb{R} \to \mathbb{R}$ and $h: \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = \frac{(3x-1)}{5}$ and $h(x) = \frac{(x^2+1)}{2}$.
State the expressions of $g \circ h$ and $h \circ g$. [20%]

IV. Let $A = \mathbb{R} - \{{}^{-9}/_{5}\}$ and $B = \mathbb{R} - \{{}^{3}/_{5}\}$. If the function $k(x)$ is defined as $k(x) = \frac{3x+5}{5x+9}$, then prove that [30%]

a) $k(x)$ is one to one and onto,
b) find a formula for $k^{-1}(x)$.

Q6.

I. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{1, 2, 3, 4\}$.

Find the following relations R_1 and R_2. [20%]

a) $R_1 = \{(x, y) | \ x \geq y\ ;\ x \in A, y \in B\}$,

b) $R_2 = \{(x, y) | x$ is odd number and $x + 3y$ is even number, $x \in A, y \in B\}$.

II. Let R_3, R_4, and R_5 be the relations on $\mathbb{Z}_0^+$, where $\mathbb{Z}_0^+$ is the set of positive integers with zero: [30%]

R_3 : x is greater than y,
R_4 : x + y = 12,
R_5 : x + 4y = 10.

Determine each of the above relations is

a) Symmetric,

b) Antisymmetric.

III. Define the equivalence relation by usual notation. [50%]
Determine whether the following relations are equivalence relations or not.

a) The relation R_6 is defined on $\mathbb{Z}$ by $aR_6\ b$ iff $a - b$ is an integer, where $\mathbb{Z}$ is the set of integers.

b) The relation R_7 is defined on $\mathbb{Z}_0^+$ by $aR_7 b$ iff $a - b$ is an integer, where $\mathbb{Z}_0^+$ is the set of positive integers with zero.

SECTION – C

Q7.

I. Simplify the following complex numbers of the form $x + iy$, where x and y are real numbers. [30%]

a) $(5 - 3i)i + i(7 + 6i)^2 + 3i^3$,

b) $\frac{(2-3i)}{(1-4i)^2}$,

c) $3 + 6i + \frac{1}{(2-i)}$.

II. Solve the equation: $z^4 + 5z^2 + 4 = 0$. [20%]

III. Let $z_1, z_2 \in \mathbb{C}$. Show that following rules: [20%]

a) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$;

b) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$.

IV. Find the principal argument and argument of the following complex numbers. [30%]

a) $z = -5i$,

b) $z = -\sqrt{5} - \sqrt{15}i$,

c) $z = 7 - 7i$

Q8.

I. If $(x + iy)^{1/3} = a + ib$, where $x, y, a, b \in \mathbb{R}$, then show that $\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$. [20%]

II. Find the complex number z satisfying the following equation:

$\left|\frac{z-4}{z-8}\right| = 1$ and $|z| = 10$. [30%]

III. Let z , a $\in \mathbb{C}$ and $b \in \mathbb{R}$. Show that $\bar{a}z + a\bar{z} + b = 0$ represents a straight line. [15%]

IV. Simplify the following expressions: [20%]

a) $(1 + i)^4 + (1 - i)^3$,

b) $\frac{13i^{18}+5i^{15}}{i+1} + 25i^{31} - 3i^{16}$.

V. Solve the equation $z^2 = \bar{z}$, where $z = x + iy$ and x, y are real numbers. [15%]

Q9.

I. ABCD is a square of which the center is at the origin. If A represents the complex number $2 + 3i$, then find: **[25%]**

a) The complex numbers which represent the other vertices of the square ABCD.
b) The lengths of a diagonal and a side of the square ABCD.

II. Let $z \in \mathbb{C}$. Draw the locus of the following equations: **[25%]**

a) $\arg(z - i) = {}^{2\pi}/_{3}$.
b) $\arg\left(\frac{1-2z}{1+\sqrt{3}i}\right) = \frac{\pi}{4}$.

III. Using the algebraic method find the locus of $|z - 1| = 2|z - 3|$ and sketch it. **[35%]**

IV. Let $z \in \mathbb{C}$. Shade the regions satisfying the inequalities $|z| \leq 4$ and $-2 < im(z) \leq 3$. **[15%]**

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